The Practical

GAUGER,

Arithmetical and Instrumental:

By Lines commonly put on Four-foot Rules; usually made for the use of the Officers in the Duty of *EXCISE.*

WITH

The full application thereof in whatsoever may concern a *Gauger* in his Geometrical affairs, for all sorts of close or open Vessels.

WITH

Plain directions to extract the Square and Cube-Root by *Arithmetick.*

AND

The Line of Proportion made more easie and familiar to any Capacity, than hitherto hath been.

By *JOHN BROWN.*

LONDON.

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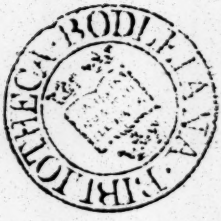
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TO

The Right Worſhipful,

George Daſhwood,
Thomas Rowney,
William Roberts,
Samuel Daſhwood,
Felix Calverd,
John Freind,
William Strong,
Edward Buckley,
} Eſquires.

FARMERS and MANAGERS

Of His Majeſties Revenue

OF

EXCISE,

Throughout *England* and *Wales*, and Town of *Berwick* upon *Tweed*;

THIS

Practical Gauger

Is humbly Preſented by

JOHN BROWN.

TO THE READER.

Courteous Reader,

HAving been imployed for ſome time, in making of Gauging Rods, of ſeveral kinds, for the Farmers and Officers of his Majeſties Duty of Exciſe; and finding ſome more ingenious than others in that Myſtery, ſome being content with the plain way by the Diagonal-Lines only; others uſing Oughtreds Gauge-Line, being a Line exhibiting the third part of the Area of a Circle, when the Diameter of Head and Bung are taken by that Line, or by Inches, and the parts of that Line juſt againſt ſo many Inches ſet down, which Line is made both for Wine and Beer-Meaſure, and now fitted with a Line of ſmaller or repreſentative Inches, whereby the whole Area of a Circle, in Wine or Ale Gallons, is as readily attained.

Other ſome uſing the way by the Gauge-Point, (as it is uſually called) with the Line of Numbers; others the Cyclometrical Tables, being the ſame with Gauge-Lines of whole Areas, or

third.

third of Areas of Circles, and by Multiplication to cast up the content of Vessels: Therefore making Rules in general for several Mens use, I have for some time made their Rules so as to fit any Mans use; that if he had no other Book or Direction about him but his Rule, he might conveniently perform any business that might fall out in the Mystery of Gauging, and that to competent exactness any way.

As first, If he used only the Diagonal-Line, and the Cask be very small, it is so graduated to shew the content to a 10th or 20th part of a Gallon; or if the Vessel be very large, as to 1 or 200 Inches, to give it to 10 Gallons expressly.

The like improvement is in the beginning part of the Gauge-Lines, to find the Area of the difference of the Head and Bung, or greater and lesser Diameters.

Also on the Line of Numbers is set the Gauge-Points for Wine and Ale Gallons, for Beer and Ale Barrels, both for round Vessels, and square Vessels also, (when desired.)

Under which Line of Numbers is set a Line of Artificial Segments, made fit to give the vacuity of an ordinary Cask, and is a Segment between that of a Cylender (or Circle,) and that for a Sphere; and therefore more fit and true for any ordinary Cask.

Lastly,

Laſtly, Over the Line of Numbers is ſet a Line of Money, or the Decimal Fraction of a Pound Sterling, or 20 *s. for the more eaſie and ready working of any Queſtion of the Rule of Three, or Practice, where Money is one of the terms.*

Of which Rule ſo contrived, I have many times been asked for a Book of the uſe thereof.

And indeed did prepare a Book about two years ſince, to have been printed with another Book, but was hindred by ſome means or other, as expecting others more able and fit to do it, that had ſeen the practical experience thereof.

But hitherto hath none appeared, that doth accommodate the mean or young Gauger to the uſe of this Rule, as I hope this will.

Therefore have I made it my buſineſs to collect and draw together this ſmall Pocket-Companion, as a help to ſome, and it may be recreation to others, I hope offence to none that love to promote Knowledge in any kind.

But I have always found by conſtant experience, that the great difficulty in uſing the Line of Numbers hath been in giving the true number of places in any product after Multiplication, and the finding the true decreaſing and increaſing of Decimal Fractions and Intigers.

To avoid which trouble I have added 2 *Prints of a Line of Numbers to* 8 *Radiuſſes, (or Revolutions,) decreaſing from* 1 *to* 00001,

and

and increasing from 1 *to* 10000, *as great a Number as can be wrought truly by any ordinary Line of Numbers, unless you use the help inserted in the Rule of Multiplication by the Line of Numbers.*

Also on the same Print is added a single, a double, a broken, and a treble Line of Numbers, for the easie and exact resolving that troublesome and difficult work of extracting the Square and Cube-Roots, often used in the Art of Gauging, *as near as such Lines can do it,* viz. *to* 3 *or* 4 *places, by inspection only, or a looking on one Line for the Cube-Number; on the other just against it is the Root to* 3 *Figures, or if you have the Root, to find the Cube to* 4 *Figures, which may prove a very good help to prevent gross mistakes in young Gaugers, or others that shall use it.*

In most of which Rules you have the way by the Pen, and also by the Instrument, the one after the other in all the cases I could remember useful, and in the briefest and plainest manner my mean skill could collect in the best method and order I could hit on, considering my inability and insufficiency for such matters, and in once writing only.

So that whether the Gauger shall use the Pen or Instrument, he may not count his Money and Time lost about this Book, wherein after I have often hinted the way of operation, the Precept

cept only ſerves without any Example, becauſe of contracting much in a little room, that the Price may be as eaſie to purchaſe as the Matter eaſie to underſtand.

I could have inſerted more variety of ways in multiplicity of words, but my deſign was uſe and conveniency.

I have all along explained the making of every Multiplicator and Diviſor as it comes in uſe, and uſed no more than may be convenient at one time or other, and ſet them together, and have not put any Tables, becauſe they are on the 4 Foot Rules as much as needs, and the Line of Numbers ſupplieth all as faſt as one can write them down, as in time the meaneſt capacity may find.

The Table of Gauge-Points, Square-Roots, Diviſors and Multiplicators, are all in a Table together, that you may ſee how they depend one upon another, to ſee the reaſon of the operations.

The three Lines of Segments are as uſeful as three Tables of Segments; and the Segments of a Sphere ſerve to find the meaſure of a Coppers crown, as in its place is ſhewed.

I have been larger than I intended at firſt, endeavouring to pleaſe all, with variety of practice, and readineſs of performance.

The working the Proportions by the Pen and Compaſſes on Numbers, is a way not hitherto uſed by any but my ſelf, and may prove of good

uſe

use to see the reason and nature of Logarithms; and a thing of good use for two Persons to compare and confirm their operations.

The second contrivance of the Line to 10 *Radiusses was thought on after the other, and the discourse not so fit for it; yet it is so plain and easie that it needs no more direction than the former: for as the other Radiusses went forward one after the other in a right Line, this lies one over the other in a shorter length, much better for carriage and performance in the use, and will shrink all alike when printed on Paper, and may be made to fold together like a Book of a competent largeness, as* 12 *Inches Radius, or the like.*

Therefore, not to detain you over long in the Preface, I refer you to the Discourse; wherein if you meet with any accidental errors, either of mine or the Printers, be pleased candidly to amend them; and if I may hear of them in the next Impression, they may be amended; for it is easie to commit, and omit to mend such oversights in these Arithmetical exercises, which I was never much exercised in. So I remain ready to serve you, in making these or any other Mathematical Instruments.

From my House, at the *Sphere* and *Sun-Dial* in the *Minories*. Dec. 8. 1677.

JOHN BROWN.

Errata in the Practical Gauger.

PAge 3. Line 36. for *eaſe*, read *moſt eaſe*. p. 8. l. 33. f. 100, r. 1000. p. 10. l. 21. add *as*. p. 11. l. 5. f. 5, r. 05. p. 12. l. 26. for *cut as*, r. *cut off*. p. 15. l. 8. f. 5, r. 15. p. 45. l. 33. f. *bo*, r. *by*. p. 51. l. 35. f. *ten*, r. *the*. p. 54. l. 12, r. *half perp*. p. 56. l. 33. f. 31.4621. r. 314.621. p. 62. l. 17. f. 31.416, r. 314.16. p. *ib*. l. 22. f. 31.416, r. 314.16. p. 69. l. 19. f. 512.620, r. 620.512. p. 89. l. 30. *dele* 800. p. 88. *the Point before the Multiplicators is to be read as a Cypher, in working by the Lines to* 11 *Radiuſſes*. p. 94. l. 14. f. 0009283.66, r. .000928366. p. *ibid*. l. 21. f. 92838, r. .000928366. p. 95. l. 4: f. *Chap*. 11. r. *Chap*. 12. p. 97. l. 18. f. *Chap*. 11. r. *Chap*. 12. p. 105. l. 6. f. *Chap*. 11. r. *Chap*. 12.

The Deſcription of the Line of Numbers, eleven times repeated, with two Radiuſſes in every Line, for the more conveniency in the uſing thereof.

1. IT is an ordinary Line of Numbers, twice repeated in one Line, after the manner of a broken Line, beginning at 3, and ending at 3.

2. Every Line is the ſame as to number of Diviſions, the uppermoſt five Lines having his ſmall Diviſions cut between two Lines only, with the fifth and tenth cuts drawn a little above the two Lines, and a Point ſet over every tenth, and three Points over every five tenth, for the better diſtinction ſake.

And drawn ſo between two Lines, to gain room for the Money-Line, and Line of Days, from 1 Day to 5 Years.

The lower ſix Lines are cut between three Lines, as is uſual for a Line of Numbers to be.

3. The Line of Money being Decimal Fractions of tenths of Farthings, of Farthings, of Pence, and Shillings, 1 *l.* ſtarling, or 20 *s.* being the Integer, begins at one tenth of a Farthing, under 0000104 in the firſt Line, which is the proper Decimal Fraction of one tenth of a Farthing, and ſo proceeds to 1 *l.* under 1 at the right end of the fourth Line.

4. The Line of Time begins at 1 Day in the third Line, and is contrived to give the Intereſt of 100 *l.* at any number of Days, from one Day to 190 Days, full half a Year, and conſequently the Simple-Intereſt of any other Sum at one operation.

Note, The Intereſt of a 100 *l.* 10 *l.* 1 *l.* is ſeen for any Days under 180, by Inſpection only.

5. Note, That when this Paper print is well paſted on a Board, it will admit of Braſs Center-Pins, in 1, 10, 100, 12, 112, 106, 17.15, 18.95, or any other place to keep it from wearing out; and for the more ready finding, and eaſie remembring of them, as the Multiplicators and Diviſors in the Table, and in uſing of them on the Lines, you muſt count the Point ſtanding before the Cyphers as another Cypher added to the Multiplicators in the Tables.

6. Some Obſervations in the uſing of it in Numeration.

Of all contrivances yet uſed, this is the plaineſt for Numeration, every Line holding forth its proper value, from a ten-thouſand part of an Integer, to 10 Millions of Integers.

As thus for inſtance:

In the middle part of the fifth Line are ſingle Integers, noted with 1, 2, 3, 4, 5, *&c.* then every longer ſtroke, or tenth, between Figure and Figure, are tenths of an Integer, and the ſhort cuts between the tenths, from 1 to 2.5, are tenths of a tenth, and from 2.5 to 5, each ſtroke is 2 tenths of a tenth, and from 5 to 10, every cut is half a tenth, or 5 parts of one tenth.

Then for the ready counting of every tenth ſtroke between the Figures, obſerve this Rule in this fifth Line, every longer ſtroke are tenths: as 2 and 5 tenths

at

at the middle, between 2 and 3, noted with three Points over the ſtroke.

In the ſixth Line right under it is 25 Integers, for between 20 and 30 is 10 of thoſe longer ſtrokes, to repreſent 10 Integers, and the ſmall diviſions between, are ſingle tenths to 25, and then every two tenths to 50, and every five tenths to 100.

In the ſeventh Line, every long ſtroke with Figures at them, are 100 of Integers; then every tenth between, are 10 *s.* of Integers, and the ſmall ſtrokes between, are ſingle Integers, from 100 to 150, and every two Integers from 150 to 500, and every 5 Integers 500 to 1000.

In the eighth, every long ſtroke with Figures are 1000 *s.* every tenth between are 100 *s.* and the ſmall between, are tens to 1500, and twenties to 5000, and fifties of Integers from 5000 to 10000.

In the ninth Line, the long ſtrokes with Figures to them, are tens of thouſands; the ten next longer between, are thouſands; the ſhort cuts are hundreds, from 10 thouſand to 25 thouſand, and 200 from 25000 to 50000, and 500 parts from 50000 to 100'000.

In the tenth Line, the long ſtrokes are hundred thouſands, the next ten thouſands; the ſhort ones thouſands, two thouſands, and five thouſands.

In the eleventh and laſt Line, every long ſtoke with a Figure at it in the middle part, is a million, as 1, 2, 3, 4, 5, *&c.* millions; the next ſhorter are hundred thouſands, and the ſhorteſt are ten thouſands, from 1 million to 2 millions and a half, and 20 thouſands to 5 millions, and 50 thouſands to 10'000'000.

Which account is readily numbred by the number of Cyphers. As thus:

In the fifth Line are no Cyphers, therefore every tenth ſtroke between Figure and Figure, is but a tenth.

In the ſixth Line, where is one Cypher, they be unites, as 21, 22, 23, 24, 25, *&c.*

In the ſeventh there are two Cyphers, and every tenth ſtroke between Figure and Figure is 10 *s.* as 210, 220, 230, *&c.*

And ſo in all the reſt, according to the number of Cyphers; as in the eleventh Line, every tenth is 100 thouſand.

On the contrary, from the fifth Line upward you muſt decreaſe as faſt.

For in the fifth Line, every Figure is a unite; in the fourth, but a tenth of an unite.

In the third Line, every ſtroke with a Figure at it, is one hundred part of an unite, or Integer.

The ſecond Line, a thouſand part; and in the firſt Line, but one ten thouſand part of a unite, or Integer, and the intermediate Diviſions to tenths, and 100 parts of what the long ſtroke with a Figure is.

7. From hence you may obſerve, that more than 4 Figures cannot well be read on this or any ordinary Line of Numbers, and what is more to be eſtimated as near as you may, and when need requires to be adjuſted by multiplying the laſt Figures by the Head or Pen, as is ſhewed before in Multiplication.

8. *To find the Simple-Intereſt of any Sum by theſe Lines by Inſpection, or with Compaſſes.*

Example.

What is the Intereſt of 1 tenth of a Pound, *viz.* two Shillings, one Pound, 10 Pounds, 100 or a 1000 Pounds, for any day under half a Year, by Inſpection?

Firſt, Seek the number of Days required among the Days, and juſt there in the Money-Line is the Simple-Intereſt of 100 *l.* for that number of Days; in the Line right over it is the Intereſt of 10 *l.* and in the Line right under it the Intereſt of 1000; in two Lines over

over it the Intereſt of 1 *l.* and in the third Line over it the Intereſt of 2 *s.* the ſame number of days.

			l.	*s.*	*d.*	*f.*10.
	1000		11	10	0	0
Example. In 70 Days,	100		1	3	0	0
what Intereſt is due	10	*l.*	0	2	3	2.5
for	1		0	0	2	3
	01		0	0	0	1.1

Thus any Sum whatſoever may be parted, and ſo the Intereſt gained by Addition and Inſpection only, as thus:

Suppoſe 125 *l.* were to have the Intereſt paid for 48 Days, what comes it to?

	l.	*s.*	*d.*	*f.*	10.
100 *l.* in 48 days demands	0	15	9	0	5
10 *l.* in like time demands		1	6	3	5
This ſet down two times and a half		1	6	3	5
For 25 *l.* the Sum added is		0	9	1	7
19 *s.* 8 *d.* Farthing 2 tenths,	0	19	8	1	2

the exact Intereſt due for 125 *l.* in 48 Days, at *6 per Cent. per Annum.*

Or ſooner with Compaſſes, thus:

The extent from 100, to right over 48 Days in the Numbers, ſhall reach from 125 to 19 *s.* 30 parts; then if 1 *l.* is but 1, 19 *s.* is 095, and the Compaſs-Point reaches to 3 tenths more, to be expreſſed thus 003, which found in the third Line, juſt againſt it in the Money-Line, is 7 *d.* 2 Farthings, in all 0 *l.* 19 *s.* 7 *d.* 2 Farthings, the Intereſt due, being near as before.

Or, This Decimal Fraction may be ſet down juſt as the Line ſheweth it, *viz.* 0.980, the 0 ſheweth 0 *l*, the 9 doubled is 18 *s*, and the 5 more is 1 *s*, *viz.* 19 *s*; then the 3 over is near 3 tens of farthings, *viz.* 30 Farthings, which is 7 *d.* half-peny, the Sum near the former, as you may ſee in *Pag.* 15, alſo in *Pag.* 40 is more of this kind.

9. For the working of compound Intereſt by the Lines, I refer you to the Triangular Quadrant, or *Kerſes Arithmetick* Pag. 322, *&c.* or in ſhort thus for a few Years.

The exact extent from 100 to 106, being laid as many times as there be Years, the ſame way from the Sum propounded, ſhall ſtay at the Principal and Increaſe in ſo many Years.

Example.

What is the increaſe of 30 *l.* 10 *s.* in 5 Years compound Intereſt, at *6 per Cent*?

The extent from 100 to 106, being repeated 5 times for 5 Years, from 30 *l.*—50, for 30 *l.* 10 *s.* ſhall reach to 40 *l.* 16 *s.* 4 *d.* the Increaſe and Principal.

Or, If you turn the ſame extent of the Compaſſes 5 times the other way decreaſing, it ſhall ſtay 22 *l.* 16 *s.* near the preſent worth of 30 *l.* 10 *s.* due 5 Year hence.

To perform this Work for many Years, Months, or Days, make use of this Table, and the Scale under the eleventh Line.

	Log. Days.	Log. Mont.	Log. Years.	
1	.00006928	.00210882	.02530586	1
2	.00013857	.00421764	.05061173	2
3	.00020786	.00632646	.05791759	3
4	.00027713	.00843528	.10122346	4
5	.00034642	.01054511	.12652932	5
6	.00041571	.01265293	.15183519	6
7	.00048498	.01476176	.17714105	7
8	.00055431	.01687057	.20244692	8
9	.00062360	.01897993	.22775279	9
10	.00069281	.02108822	.25305865	10
11	.00076215	.02319704	.27836452	11
12	.00083141	.02530586	.30367038	12

These Numbers are the Logarithms for 12 Days, 12 Months, and 12 Years, and by Addition, or by Multiplication, you may find the Logarithm of any time whatsoever.

Example.

What is the Increase or Rebate of 53 *l.* 5 *s.*, in 15 Years, 9 Months, and 17 Days?

In the foregoing Table	
The Logarithm for 10 Years is	.25305865
The Logarithm for 5 Years is	.12652932
The Logarithm for 9 Months is	.01897993
The Logarithm for 10 Days is	.00069281
The Logarithm for 7 Days is	.00048498
The Sum of all these Logarithms is	.39974569

Then

Then this extent taken from the Scale of equal parts, counting the first Figure 3 just under 800·000 in the eleventh Line; then for the first 9, count 9 tenths of the first part before 1; then for the second 9, count by estimation 9 tenths of the last tenth more; then for the 7, that is to be clearly estimated, being in all very near 4 of the great parts.

This extent so taken between the Compasses, is the exact extent for 15 Years, 9 Months, and 17 Days, on this Line of Numbers, and being laid from 53.25, for 53 *l.* 5 *s.* increasing, gives 133 *l.* 13 *s*; or being laid the other way decreasing, gives 21 *l.* 4 *s.* 6 *d*, the present worth of 53 *l.* 5 *s.* due 15 Years, 9 Months, and 17 Days to come, at *6 per Cent.* Compound-Interest.

10. For the increase of present worth of Annuities, at *6 per Cent.* First find what Sum of Money hath the Interest thereof, equal to the yearly Rent, half year, or quarterly Rent, or Payment. Thus:

Suppose the Rent be 10 *l.* a Year. Say

As 6 to 100; so is 10 to 166 *l.* 666.

Then what shall the increase of 166 *l.* 666 be in 21 Years?

The two Logarithms of 10 Years, and 11 Years added, is .53142317, and being taken from equal parts, and laid increasing from 166.66, gives 566 666, from which Sum when 166.666 the first principal is subtracted, remains 400 *l.* for the Increase, or Rent due to be paid at the end of 21 Years, having received none all that time.

11. And the same extent laid decreasing from 400, shall reach to 117 *l.* 12 *s*, the ready Money that shall purchase a Lease of 10 *l.* a Year, to continue 21 Years, and pay no yearly Rent.

12. In

12. In uſing theſe Lines, always ſet the points of the Compaſſes in that Line of the three Marginal-lines, (between which the tenths are cut) that the Braſs-Center-Pins are put in, which is always the lower-moſt (or the middle) Line; for, elſe when you meaſure acroſs, you can have no true anſwer.

13. In finding the Square-Root of a Number when the Figures be odd, the left hand 1 is the unite, and when they be even, the right hand 1 is the unite, or elſe you cannot divide the ſpace between the number and 1 into two equal parts, when you meaſure acroſs the Lines.

14. 1. In finding the Cube-Root of a Number when the Point falls on the laſt Figure, the left hand 1 is unite.

2. When it falls on the laſt but 2, then the right hand 1 is unite.

3. But when it falls on the laſt but 1, then any of the ones, either the right or left may be unite; but in turning the Compaſſes three times, you will paſs by one of the unites, or elſe you cannot divide the ſpace between 1 and the number given into three equal parts, as it muſt be in this operation, according to one general rule, and the firſt of three repeatings from 1, is the Cube-Root required.

Therefore in this caſe, if your Number be in the right end, the left hand 1 is the unite; but if your number be in the left end, then the right hand 1 is the unite.

Example of the three Rules:

1. To find the Cube-Root of 1953125, where the Point falls on the laſt Figure, the unite will be on the left hand of the Number.

For

For one third part of the extent, between 1 in the fifth Line, and 1953125 in the eleventh Line, will fall exactly on 125 in the seventh Line, which number 125 is the exact Cube-Root.

2. When the Point falls on the last Figure but two, as in 941192, or 571787, then the unite will lie on the right hand of the Number.

Thus the exact third part between 1 on the right end in the fourth Line, and 941192 in the tenth Line, will be at 98 in the sixth Line.

Or more apparently: The exact third part between 1 in the fourth Line, and 571787 in the tenth Line, will be at 83 in the sixth Line, one Line being between every extent of the Compasses, which could not be if you went from this number in the tenth Line toward the other 1 at the left end.

But note, that you may work it by counting the 571787 in the beginning of the eleventh Line, and the unite of the fifth Line, for there also is six Lines between the number and 1, to be parted into three equal parts, yet the unite lies on the right hand of the Number here also, as it did before.

The like may happen in the first way also, when the Number falls between 1 million and 3 millions; or any first Figure of a Number, being between 1 and 3, where the Rule is the same, the Unite being on the left hand of the Number propounded.

3. But when the Point falls on the last Figure but one, then the first Figure between 3 and 9, the right hand 1 is the unite, and the left hand unite lies between the Number and it.

But when the first Figure of the Number propounded is less than 3, you must count or find it at the right end of the Lines, and the left hand 1 is the unite, and the right hand 1 is between the Number and it.

Example

Example both ways.

If this Number 28372625 be propounded to find the Cube-Root of it, this Number is found in the eleventh Line at the right end, and the exact third part between the left hand 1 the Number, is at 305 in the seventh Line.

Again, If 62431 were given to find the Cube-Root of, it would be found at the left end of the tenth Line, and then the unite muſt be at the right end, on the fourth Line; for, otherwiſe you cannot divide the ſpace between into three equal parts, as the general Rule is.

Thus the exact third part, between 1 in the fourth Line, and 62431 in the tenth Line, is at 39.7, the Cube-Root required.

Note always, that the firſt of the three parts, counting from 1, is the Cube-Root.

Thus I have been over large to make this matter plain to any capacity.

Some Examples in the Rule of Three, and Practice, by theſe Lines.

1. *If* 1 *coſt* 3 *l.* 10 *s.* *what coſt* 112?

The extent from 1 in the fifth Line, to 3.5, for 3 *l.* 10 in the ſame Line, ſhall reach the ſame way from 112 in the ſeventh Line, to 393. and 0163 more, which 0163, ſought on the fourth Line of Numbers, on the Money Line, juſt under it is 3 *s.* 3 *d.* the near anſwer, being in all 393*l.* 3 *s.* 3 *d.*

2. *If* 112 *l.* *coſt* 4 *l.* 10 *s.* *what coſt* 1?

The extent from 112 in the ſeventh Line, to 4 *l.* 10 *s.* in the fifth Line, ſhall reach the ſame way from 1 in the fifth Line, to 9 Pence, half-Peny, and half-Farthing, and better in the third Line.

3. *If*

3. *If* 3 *Yards and* 3 *Quarters of Riboning coſt* 9 *Pence* 2 *Farthings, what ſhall* 17 *Yards and a Quarter coſt?*

The extent from 3.75 in the fifth Line, to 9 Pence 2 Farthings in the fourth Line, ſhall reach the ſame way from 17.25 in the fifth Line, to 3 *s.* and 8 *d.* in fourth Line, the anſwer to the Queſtion.

Where you may always obſerve, that the ſame extent you take from the firſt Number to the ſecond, counting the Lines between, the ſame extent counted the ſame way, and the ſame number of Lines, muſt be between the third and the fourth Number, in all operations whatſoever.

FINIS.

THE Practical Gauger.

CHAP. I.

The Deſcription of the Rule or Gauging-Rod, in three Joynts.

FOR this, few words may ſerve the turn, being partly deſcribed in the Preface to the Reader. The uſual way of making them, is a Four-foot Rule in three Joynts, and four Pieces, braſed at the heads to keep it from cleaving at the head, made uſually of one bigneſs, *viz.* about 75 or 80 of an Inch ſquare when ſhut together, or 37 or 40 of an Inch ſquare being open, to the intent that two or three ſuch Four-foot Gauging-Rules may be faſtened together, to take Diameters of large open Tuns, by means of one or two Sockets, ſuch as I have made for that purpoſe, being the moſt portable of any other way.

The Lines deſcribed on theſe Rules, are,

1. On one outermoſt ſide the two Diagonal-Lines, one for Wine, the other for Beer or Ale Gallons, to 282, (or ſometimes (which I count the beſt way,) Inches in ten parts, and one Diagoanl-Line only, for Beer or Wine, which you pleaſe, counting 232 Inches for Wine, or 282 for Beer Meaſure, to be the ſolid content of the Beer Gallon.)

2. On the next outside free from the heads, is the Line of Inches in ten parts, from 1 to 48, and next it the Line called *Oughtreds* Gauge-Line, (but if you have but one Diagonal-Line, then on this side is set two *Oughtreds* Gauge-Lines, one for Beer Gallons, the other for Wine, which I hold of most general use.)

3. On the other side and as near *Oughtreds*-Line as may be, is set the Line of small Inches.

4. On the other side for half way, is set the Lines of Numbers, Segments, and Pence, being Decimal Fractions of 20 Shillings in Farthings and Pence, from 1 Farthing to 2 Shillings, a tenth of 20 Shillings.

This is the usual work put on the 4 sides of the Ruler, yet so as that room is left for a Line of 8 Radiusses, and particular Lines of Ullages, if any should desire them to be put on. All which Lines have their names set to them, on some part of the Line so called, which may be some help to the Learner.

Also the two Diagonal-Lines, and the two Gauge-Lines, being one for Wine-Measure, and the other for Beer, are set always together on one side, that you may not be troubled to look for the name, to know which is the one or other of them, for the Beer Gallon being greater than the Wine in bulk or solidity, the least Number on the Rule at any Diagonal or Diameter is always the Beer Gallon, as by use you will plainly see.

The Lines on the printed Plate in the Book are only the Lines of Numbers, Pence, and Segments, to 1, 2, 3 and 8 Radiusses, and one Line of Time joyned to that of 8 Radiusses for Interest-operations, and every Line having the name graven upon it needs no further description, nay, is better than many words can illustrate.

These Lines have been set on Sliding-Rules, as is to be seen in a Book of *Gauging*, set forth by Mr. *John Smith*, in the Year 1673.

The

The use of which is (neer) the same as the Four-foot Rule is, to be used with Compasses if you please, or without, by sliding one by the other, as will soon be perceived.

Moreover, since the first writing of this Discourse, I have drawn another Line of Numbers to 10 Radiusses, a print of which is joyned to the Book, whose use is the same with the other, only as in one, the extents went forwards in one continued Line, this leaps over one another, as it were, and the observation is, to mind how many Lines is between the first and second terms in the question: to have so many between the third and fourth, and if 5 Radiusses be too little to express your Number, or if the Compasses fall off the Lines by counting so many Radiusses, then so many Cyphers as you increase the third Term, so many must you increase the fourth.

As in this Multiplication the extent from 1 to 2346, there is two Lines between the first and second, and the same extent laid from 1279 and two Lines counted between, then the Compass-Point will fall over the lower Line. In such a case, remove the third Point a Line or two higher, calling 20, 2000, then as you add two Cyphers to 20, so you must add 2 Cyphers to 20000, and it becomes 2000000, or 2 Millions; and the answer is 3000534, but to 6 places is enough for any Instrument.

1279
2346
3000534.

The use of the Lines, and first of the Line of Numbers.

The Line of Numbers, known by the name of *Gunters*-Line, is a Line of Proportion Geometrical, whereby is performed Multiplication, Division, Rule of three, in single, double, or treble Proportion: Practice, Extracting of the Square or Cube-Roots, or higher Powers, with the least trouble and ease of any other way yet in use.

And though it will not afford that certainty and exactneſs in theſe Problems as is many times requiſite, yet it is uſeful, and of great ſervice to all ſorts of Mechanical Men, and may prove a ready help, and as a proof to more able Artiſts and Penmen, and ready Arithmeticians.

However, the moſt general concerns of *French*, *Spaniſh*, Brandy, Wines and Oyls, brought to the Port of *London*, are caſt up by the Line of Numbers, and a reſolution of the quantity of Gallons preſently ſtated, by a motion of the Sliding-Rule only, where the content in Gallons to half a Gallon, is as near as is to be expected, the beſt Artiſt not preſuming to approach nearer, becauſe of the difficulty and uncertainty in getting the true dimenſions of thoſe inviſible inſides of cloſe Casks.

This Line of Numbers that ſome hundreds know, and many thouſands do not know, ſeveral ſmall Books have been write concerning it, to which I might refer you, but that were to put you to trouble; therefore as plainly and as briefly as I can to the uſes thereof.

PROB. I.

Numeration on the Line of Numbers.

This firſt Problem is the moſt difficult to Learners or young Beginners in this Myſtery, and therefore is to be better heeded, being like knowledge of Letters to Reading.

Therefore, on the Line of Numbers ſet on Gauging-Rods, being a Line to two Radiuſſes or Revolutions, the Figures are uſually on Four-foot Rods, thus.

At the beginning of the end (not braſed,) is 1, then forward with 2,3,4, *&c.* to 10 at the Joynt, then with 20,30 40,50, *&c.* with 100 at the end cloſe to the head of the Joynt, whoſe proper power or name may be as it is numbred, from 1 to 100, then the intermediate

diate diviſions between, in the firſt part from 1 to 10, are tenth parts of one Intiger, and note, that from 1 to 5 you have tenths, and half tenths, and afterward from 5 to 10, every whole tenths only.

And between 10 and 20 in the ſecond Radius, the longer cuts cut croſs one Line, and between two are Intigers, as thus 10,11,12,13,14 and 15 being cut up higher, then the 11,12,13 and 14 is for the ready finding and counting them, and the little ſhort cuts cut between two Lines, are tenth parts of Intigers, and ſo it proceeds to 20.

After 20 every ſhort cut is two tenths, and ſo it proceeds to 40, and from 40 to 100, every ſhort cut is half an Intiger, or five tenths of an Intiger. Thus the Line is read in its plain apparent account, but when you deal with a greater Number than 100, then 10 at the middle of the Line of Numbers is to be called 100, and then 20 is 200, and every ſmall ſtroke between are ſingle Intigers, from 10 to 20 there being 100 parts between 10 and 20, now called 100 and 200, and every ſingle Intiger before, is now become tens of Intigers.

But if you make uſe of four Figures, then 10 in the middle is called 1000, and 20 called 2000, and 100 at the end 10000, (beyond which this Line cannot go to be perceived,) then by conſequence, if the middle 10 be called 1000, 1 at the beginning muſt be called 100, and the 5, 6, 7, 5, 6 and 7 hundred, and every ſmall cut between 5 and 10 tens of Intigers. This being minded and duly conſidered, which is reading of the Line, the uſe is

PROB. II.

To find out any Number under 5 Figures, on the Line of Numbers.

Seek the firſt Figure of the Number propounded, among the longeſt ſtrokes with Figures at them.

For the next Figure of the Number propounded, count ſo many tenths forward as that Figure contains Units.

For the third Figure of the Number, count ſo many tenths from that laſt tenth forwarder as that Figure hath Units.

For the fourth Figure of the Number propounded, count ſo many tenths forward from the laſt Centiſm by eſtimation, and that ſhall be the exact Point to repreſent the Number propounded, as in theſe three Examples.

To find the Point repreſenting 12, *on the Line of Numbers.*

Firſt, For the Figure 1, the firſt Figure of 12 the Number given, you may count either at 1 at the beginning, or at 10 in the middle; then for 2 the next Figure of 12 the Number given, count from 1 ſo many tenths between 1 and 2, or 10 and 20, and that ſhall be the exact Point to repreſent 12, the Number propounded to be found out on the Line of Numbers.

2. Again, to find 231 the Cube-Inches in a Wine Gallon.

Firſt, For 2 the firſt Figure, count two in the firſt part, or 20 in the ſecond.

Secondly, For 3 the next Figure, count three tenths forward from the ſtroke 2 or 20.

Thirdly. For 1 count 1 Centiſm more, from the laſt three tenths which here is at half a part, every part repreſenting two tenths.

Again, to find 17, 15 the Gauge-Point for a Wine Gallon, or if it had been 1715, the Point here on the Numbers is at the ſame place firſt, the 17 is between 10 and 20, then for the 15 count one tenth and a half beyond 17, and that is the exact Point repreſenting 17, 15, or 1715, where on the Gauging-Rods is a Braſs Center-Pin uſually put, to find the Gauge-Point more readily

readily, and to keep it from growing too wide with often putting the Compass-Points in one place.

Note, such like Points you have at 107, 19 the Gauge-Point for an Ale Barrel, and at 113, 69 the Gauge-Point a Beer Barrel, noted with A B and B B, neer to 10 on the Brass.

I have used many words in this that I might be plain, and when this is easie and well observed, then nothing can be hard: But yet the great difficulty will be what to call the Product after a Multiplication, or what a Quotient after Division, or a fourth Number in the Rule of Three.

And to make this doubt infallible and certain, I have added a print of the Line of Numbers to 8 Radiusses, when the print is cut in the middle, and 1 at one end pasted exactly on 1 at the other end, then 1 is in the middle, and toward the right hand you have 4 Radiusses more than 1, and to the left hand 4 Radiusses less than 1, and here every Radius bears his own proper value, and needs no rule or direction to know how to call it.

Yet with this Proviso, that in Numbers above 100 you must estimate the Intigers, and after 1000 you must estimate the tens, because the Line is of a short extent or Radius.

To remedy which inconveniency, another contrivance came in my mind, which is here in a brief way also inserted, with the several Radiusses one over another; which on a round Board or Plate is drawn to 3 Foot Radius, and here you must number the turns between the first and second Term, to have so many between the third and fourth.

Also note, that a small pair of Compasses will reach over 4 or 5 Radiusses, though it be a Radius of 6 Inches: Also by lying right over one another, the shrinking of the Paper will be as much in one patt as in another, and therefore the exacter in the use.

Note

Note also, having 2 Units you may work the Square and Cube-Roots, using that 1 which admits of halving or thirding the Space between 1, and any Number that can be propounded to 9 places.

2. In the Line of Money next to it, you have the 25*th* part of a Farthing in the first Radius to 1 Farthing, or the 100*th* part of one Penny, and after 1 Farthing every fifth part of a Farthing, or 20*th* part of a Penny to 1 Penny, and after 1 Penny only every Farthing to 2 Shillings, or 1 tenth of a Pound Sterling, and from 2 Shillings to 1 Pound, only every 3 Pence as by considering the parts between every Number certainly expressed will be apparent enough.

This Line of Money ends at 1 Pound, for then every Intiger on the Numbers is 2 Shillings to 10 Pound, and afterwards every Intiger is 1 Pound to 100.

Then after 100 every Intiger is 10 Pound, and in the last whole Radius every Intiger is 100 Pound, and need to be exact in the estimation of the parts, and here is plainly seen the defect of Instrumental Arithmetick in great Numbers above 4 Figures, wherein you may use a greater denomination, as Feet instead of Cube-Inches, and Barrels instead of Gallons, being as near as is required at the end or resolution of the demand or question.

3. The Line of Time begins at 1 Day (parts of a Day being inconsiderable in questions of Interest) and so it proceeds to 60 Days or 2 Months, then every 4*th* part of a Month to 1 Year, and after 1 Year to quarters only to 20 Years, and after 20 Years every single cut is a Year to 1000 Years.

Lastly, After 100 Years, every cut doth doth represent 10 Years, and so to 100 Years.

Here may you Geometrically see the vast increase of Geometrical-Proportion continued, where after a few turns it exceeds all visible sight, and in time (with a few more repetitions) all intellectual conceptions or intellegible reading in Numbers: And if you count

the

the other way from 1, you have the ſame proportional decreaſe, *viz.* to the 1000*th* part of an Intiger, at the firſt 00001 at the beginning of the Line to 8 Radiuſſes.

This will prove an excellent help to Learners, and a ſettling of the Mind to able Proficients, in many a doubtful matter wrought by a ſingle Line only, and one general Rule in many caſes ſerves, without exception, as will be hinted in place convenient, in the following diſcourſe.

CHAP. II.

Of Proportion, or mean Proportions.

MAthematical Authors make mention of three mean Proportions between Numbers, *viz.* Arithmetical, Geometrical and Muſical.

Arithmetical Proportion, is the half ſum of the two extreams added together, as thus, the Arithmetical mean between 25 and 14 is $19\frac{1}{2}$, being the half of 39 the ſum of 25 and 14 added together.

The Geometrical mean Proportion, is the Square-Root of the product of the two extream Numbers, multiplyed one by the other, thus, a mean Proportion Geometrical between 25 and 14, is $18\frac{7}{10}$ the Square-Root of 350 the product of 25 multiplyed by 14, which Square-Root is found by dividing the ſpace on the Line of Numbers, between 350 and 1 into two equal parts; but to do it by the Pen, requires many words to ſhew the way how, not fit for this place.

The Muſical mean Proportion, is leſs than the Geometrical, and thus found. As the Arithmetical mean to one extream, ſo is the other extream to the Muſical mean; for if you multiply 14 by 25, the product is

350,

350, then 350 being divided by 19, 5, by adding Cyphers to 350, you ſhall find the quotient to be 17, 95, the Muſical mean required. But by the Line of Numbers thus, the extent of a pair of Compaſſes from 19, 5 to 14 being laid the ſame way from 25, ſhall reach to 17, 95 as before.

The Geometrical mean Proportion between two Numbers, is moſt in uſe and found by Arithmetick as before, but by the Line of Numbers, moſt eaſily thus, the middle between one extream and the other, meaſured on the Line of Numbers, is the Geometrical mean required, thus the exact middle btween 25 and 14, is 19, 5, the Geometrical mean Proportion required.

Alſo note, if you divide the diſtance between the two extreams, into 2, 3, 4 or 5 parts, every extent ſhall be a Geometrical Proportion, or continual mean Proportions Geometrical, as 1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, *&c.* by taking the exact extent between 1 and 2 on the Line of Numbers, and repeating it orderly from 1, 10 times it ſhall reach at laſt to 1024 done by the Pen, by a continual multiplying of 1 by 2, the conſtant difference; for two times 1, is 2; two times 2, is 4; two times 4, is 8; and two times 8, is 16, *&c.*

Or, Geometrical Proportion may be diſcontinued, in moſt queſtions of the Rule of Three. As thus,

As 8 to 12, ſo is 19 to 28, 50.

By the Pen, thus, multiply 12 by 19, and the product is 228, to which adding of two Cyphers to continue the Diviſion divide by 8, and the quotient is 28 50 the fourth proportional required, and thus is the Rule of Three in Arithmetick by the Line done thus, the extent from 8 to 12 being laid the ſame way from 19, reaches to 28.

So alſo it may be continual, or diſcontinual Proportion decreaſing, as the two former Examples were increaſing

inceraſing. *Example*, As 1 to 05 which is half of one, ſo is 05 to 025 which is a quarter of one.

The extent from 1 to 05 ſhall reach the ſame way from 5 to 025, or as 12 to 8, ſo is 6 to 4, for if 8 be multiplyed by 6, the product is 48, and 48 divided by 12, quotes 4.

CHAP. III.

Multiplication by the Line of Numbers, or Line of Proportion.

THe extent from 1 to the Multiplicator being laid the ſame way from the Multiplicand, ſhall extend to the product.

Example. Suppoſe I ſhould multiply 12 by 30, then the extent from 1 to 12 being laid the ſame way from 30, reaches to 360, the product required.

Or you may ſay as 1 to 30, ſo is 12 to 360, for note, that number which is neareſt or moſt convenient to take from 1 is beſt to make the ſecond term, whether it be the Multiplicator or Multiplicand: It is no matter which you count the ſecond term, 1 in Multiplication being always the firſt, and the product the laſt term.

Note alſo, that there muſt be as many Figures in the product as in the Multiplyer and Multiplicand being put together, except when the two firſt Figures of the product are greater in value than the two firſt Figures of the Multiplicator or Multiplicand, then it conſiſts of one Figure leſs.

Example. To multiply 85 by 96, the product is 8160 of four Figures, becauſe 81 the two firſt Figures in the product are leſs than 85 or 86 the two firſt in the Multiplicator or Multiplicand.

Again,

Again, to multiply 231 by 16, the product is 3696 of four Figures only, because 36 the two first Figures of the product are greater than 23 or 16.

Note lastly, that in regard you cannot see above four figures at any time, nor them always certainly, observe this brief Rule.

Set the Numbers down as in order to multiply them by the Pen: As for Example, let 561 be multiplyed by 32, say two times 1 is 2, and two times 6 is 12, set down 2 and carry 1, *&c.* then 3 times 1 is 3, *&c.* then by addition, say 2 is 2, and 3 and 2 is 5, thus have you gained the two last Figures which you could not see on the Rule, and the whole product is 17952. First, of five Figures, because 17 is less than 32, and the 179 the Line sheweth, and the 52 you have by the last Rule, thus you must help the defect of the Rule in great sums when you multiply a whole Number and a Fraction together: Note, that the number of places as to Intigers, must be ruled by the Intigers only and not by the Fractions.

Example. Let 3 Foot 75 Parts be multiplyed by 7 Foot and 80 Parts, the Numbers are set down as whole Numbers thus, and multiplyed as whole Numbers; but after Addition you are to cut as many Figures as there be Fractions in the Multiplicator and Multiplicand, as in this Example 4 Figures for 7 5, and 8 0.

·3 75
·7 80
30000
2625
29·2500

Also, when you multiply a Fraction by a Fraction in Decimals, you must be sure to keep the Figures in their right places.

Example. If 0072 be multiplyed by 075, the Numbers are set thus, and multiplyed altogether as Decimal or whole Numbers, and yet the product instead of increasing is diminished, as doth most plainly appear, by working by the Line to 8 Radiusses, for the extent

.0072
.0750
3600
00505
.0054000

extent from 1 to 0750, laid the ſame way from 0072, reaches to 0054, a number leſs than 0072, the leaſt of the other two Numbers.

CHAP. IV.

Diviſion by the Line of Numbers.

THe extent from the Diviſor to 1, (being counted at 1 at the beginning, or at 10 in the middle, or 100 at the end) ſhall reach the ſame way from the Dividend to the Quotient, conſiſting of as many Figures as the Diviſor may be ſet times under the Dividend, in the working by the Pen.

And note, that the Fraction remaining is always a Decimal Fraction, and is reduced to the Vulgar Fraction given by the Pen, by turning the ſame extent the contrary way from the Decimal Fraction, reduceth it to the Vulgar Fraction.

Example. Let 595 Beer Gallons be divided by 36, to find how many Beer Barrels is in 595 Gallons.

The extent from 36 to 1 being laid the ſame way from 595, gives 16 Beer Barrels, and the Decimal Fraction over, is 527: then the ſame extent laid the contrary way from 527, gives 19 Gallons, the Fractions over as will come forth, when you ſhall divide by the Pen.

Again, In a greater Sum, let 52197 Shillings be reduced to Pounds by Diviſion on the Line.

The extent from 20 to 1, being laid the ſame way from 52197, ſhall give 2610 Pounds near.

Note here, the Quotient is known to conſiſt of four Figures, by the Rule laſt noted, but to give the exact anſwer, is not to be done certainly, without breaking the Number, thus:

First, The extent from 20 to 1, being laid the ſame way from 50000 Shillings, give 2500 Pounds: then the ſame extent laid from 2000 Shillings, gives 100 Pounds.

Laſtly, The ſame extent laid from 197 Shillings, gives 9 Pounds 17 Shillings; then theſe three Sums found by three extents added together, make 2609 Pounds 17 Shillings, the exact anſwer.

CHAP. V.

Reduction by the Line of Numbers.

REduction by theſe Lines is performed by inſpection for the moſt part, from Pounds, Shillings, and Pence to a Decimal Fraction, and the contrary: As thus for Example.

I would reduce 7 Shillings 6 Pence to a Decimal Fraction: On the Line of Money that is joyned to the Line of 8 Radiuſſes, ſeek for 7 Shillings 6 Pence, and juſt againſt it on the Line of Numbers is 0375, the Decimal Fraction required, 1 Pound, or 20 Shillings being the Integer.

Or, to find the ſame by the Numbers on the Gauging-Rod, Note, that if 100 at the Joynt be called 1 Pound, or 20 Shillings, then 7 Shillings 6 Pence is at 0375 backward; for if 100 be 20 Shillings, then 9 is 18 Shillings, 8 is 16 Shillings, ſo 3 is 6 Shillings, and 35 is 7 Shillings, and 375 is 7 Shillings 6 Pence. Then, in regard that 100 is but 1, 35 is leſs than 1, therefore 0375, as was plainly expreſſed on the Line to 8 Radiuſſes.

On the contrary, to bring a Decimal Fraction to Money, ſeek the Decimal Fraction on the Line of Numbers, and juſt againſt it on the Line of Money, is the anſwer required. *Example.*

Example. Let 00729 be reduced to Money, just against 00729, found on Numbers, on the Money-Line, is 16 Pence 3 Farthings, the true Answer.

Again, Suppose I would reduce 5 Pounds 15 Shillings 6 Pence to a Decimal Fraction.

First note, the 5 Pounds is 5 Integers; then if you look for 5 Shillings 6 Pence on the Money-Line, on the Line of Numbers is 0775, which added to 5, the Integer makes 5.775, the Point standing instead of 0. after the 5 and before .775.

Or, without the Lines thus, by the Pen: let 59 Pounds 8725 of 10000 part of a Pound, be reduced to Money. First, the 59 is 59 Pounds. Secondly, every Unite in the first Figure 8 is 2 Shillings, therefore 8 is 16 Shillings; then 5 in the second Figure next to 8 is 1 Shilling: therefore 7 is more than 1 Shilling. Note, the remainder above 5 in the second place, is tens of Farthings, and the third place is Farthings, and the fourth place tenths of 1 Farthing: So that the whole Fraction is 17 Shilling 5 Pence 2 Farthings very near.

But for any other reduction, the Rule is always thus, as one Denominator to the other Denominator, so is the given Numerator to the inquired Numerator.

Or, as one Denominator to his Numerator, so is the other Denominator to the inquired Numerator.

Example. In 154 Yards, how many Ells? the extent from 5 to 4, because 5 quarters of a Yard make 1 Ell, shall reach from 154 to $123\frac{1}{5}$ the number of Ells (contained in 154 Yards.)

By the Pen, thus: 154 multiplied by 4, the quarters in a Yard, and the Product divided by 5, the quarters in an Ell, produceth 123 Ells and $\frac{1}{5}$ the reduction required.

Or, if you had multiplied 154 by 36, the Inches in a Yard, and divided the Product by 45, the Inches in 1 Ell, quotes 123 Ells, and 9 Inches over.

CHAP. VI.

To find the Square-Root of a Number by the Pen.

1. THE extraction of the Square-Root of any Number, is to find out another Number, that being ſquared, or multiplied by it ſelf, the Product is equal to the firſt Number propounded.

Thus, if 900 be a Number propounded to find the Square-Root of, you ſhall find 30 is the Square-Root thereof; for 30 ſquared, *viz.* multiplied by 30, is 900 the Number firſt propounded.

And the invention to find out 30, is called the extraction of the Square-Root.

2. Square-Numbers are ſingle or compound: ſingle, whoſe Roots conſiſt of one Figure: or compound, whoſe Roots conſiſt of many Figures.

The ſingle Square-Numbers are only theſe 9 under a 100, and the ſingle Digits from 1 to 9 are the Square-Roots of theſe Square-Numbers, and uſed, as the Multiplication Table, by heart.

All others above 100 are compound Square-Numbers.

Root.	*Squar.*
1 .	1
2 .	4
3 .	9
4 .	16
5 .	25
6 .	36
7 .	49
8 .	64
9 .	81
10 .	100

3. All Numbers are either right Square-Numbers, whoſe Roots ſquared come to the ſame, without any Remainder, as 64, whoſe Square-Root is 8, and 8 ſquared, is 64; or 144, whoſe Square-Root is 12, for 12 times 12 is 144.

Or

Or else called **Surd-Numbers**, whose Roots being squared will not come to the first Number without some Fraction or Remainder: thus 70 is a Surd-Number, whose nearest Root is 8, and 367 of a Decimal Fraction more.

4. When you would find the Square-Root of a Number, you must prepare it with Points or Periods, thus: put a Point under the first Figure, the third, the fifth, the seventh, and ninth, counting from the right hand toward the left, missing one, and pointing the other: And observe, that so many Points as the Square-Number hath, so many Figures will be in the Square-Root thereof.

Thus this Number 127351225 hath 5 Points or Periods, and his Square-Root, *viz.* 11285 hath 5 Figures.

5. The Number being so prepared, observe the Rule in these four Examples.

First Example is to find the Square-Root of 5329.

Set the Number down, and point it, and draw a Quotient-Line at the end, as in Division, (which this Rule is like) only in Division the Divisor is given, here you are to seek it. Again, in Division one Divisor is used, here at every Period you seek a new Divisor.

Then to find the first Divisor, demand what is the greatest Square-Number, in 53, the Figures of the first Period, and you shall find it to be 49, as in the little Table aboveſaid, whose Square is 7.

Set 7 in the Quote, and the Square of 7, *viz.* 49 under 53, and subtract just as in Division, and cancelling 53, set 4 the Remainder right over 3, as in the work you see.

Then for the second Period, draw a Line under 49 as far as the next Period.

Then double 7, the Figure in the Quote, which is 14, set 4 the last Figure of 14 under the Line, and also right under 2, the first Figure of the second Period, and the 1 backwarder under 4, the remainder of the first Period, more to the left hand.

```
  0
  0400
  5329 (73
  49   the Square of 7.
  143  the double of 7.
  429.143 mult. by 3.
```

Then to find the next Divisor, demand how many times 14 may be had in 42, the Figures right over it being the remainder of the first Period, and the first Figure of the second, and the answer is 3, just as in Division; being careful not to take too much or too little: which 3 set in the Quote, and also after the 14, under 9 the last Figure of the second Period, then you have 143 set under the Line.

Then multiply 143, by 3 set in the Quotient, and it comes to 429, after the rules of common Multiplication.

Then substract 429, from 429 right over it, being the remainder of the first Period, and the two Figures of the second Period, and the remainder is 0, because it is a just Square compound Number; for 73 squared, comes to just 5329, the Number first given.

A second Example let be 23.145721, *whose Square-Root is required.*

1. Set down the Number, point it, and draw a Quote-Line at the right end, as in the Example you see.

Then for the first Divisor, demand what is the greatest Square-Number in 23 the first Period, and the Tablet saith 16, whose Root is 4; set 4 in the Quote, and the Square of 4, *viz.* 16, right under 23; then substract

ſubſtract 16 from 23, cancelling 23, and ſetting the remainder right over, as you do in Diviſion.

2. Then draw a Line under 16, as far as 4 the laſt Figure of the next Period. Then double 4 the Figure in the Quote, which is 8; ſet 8 under the Line, and right under 1, the firſt Figure of the next Period, *viz.* 14.

Then to find the next Diviſor, demand how often 8 may be had in 71, the Figures right over it being the remainder of the firſt Period, and firſt Figure of the ſecond Period, juſt as in Diviſion, and the anſwer is 8. Put 8 in the Quote, and alſo after 8 under the Line, and right under 4 the laſt Figure of the ſecond Period

```
00000
07x09800
23x4572x (4811
16 the Square of 4.
  88 | double 4 more 8.
 704 | 88 multiplied by 8.
 961 double 48 more 1.
9621 double 481, & 1 added.
```

Then multiply 88, by 8 laſt ſet in the Quote, and the Product is 704; then ſubſtract this 704 from 714, the Figures right over them, and the Remainder is 10, as in Diviſion.

3. Then draw a Line under 704, as far as 7 the laſt Figure of the third Period, and double 48 the Quote which makes 96. Set 6 under 5 the firſt Figure of the next Period, and the 9 backwarder, then demand how often 96 may be had in 105 right over, and the anſwer is 1, ſet 1 in Quote and after 96, which makes it 961, then 961 multiplied by 1, the Figure laſt ſet in the Quote, and it is but 961; then 961 taken from 1057 right over it, remains 96, cancelling 1057, as in Diviſion.

4. Then draw a line under 961 as far as the laſt Period, and double 481 the Quote, which makes 962, putting

2

2 right under 2 the firſt Figure of the laſt Period; then demand how often 962 may be had in 962, the Figures right over them, and the anſwer is but 1.

Set 1 in the Quote, and alſo after 962, and it makes 9621.

This Number multiplied by 1, the Figure laſt ſet in the Quote, makes 9621. Then this 9621 ſubſtracted from the ſame Figures right over, which Subſtraction being made, remains 00: Therefore the Number 23149721 is a right Square compound Number, and 4811 is the exact Square-Root of it.

A third Example let be 87135

Set the Number down, point it and quote it, as in the Example.

1. Then demand what is the greateſt Square-Number in 8, and the Tablet ſaith 4, whoſe Root is 2; ſet 2 in the Quote, and the Square of 2, *viz.* 4 right under 8, which taken from 8 remains 4, cancel 8 and ſet 4 over as in Diviſion.

2. Then draw a Line under 4, as far as 1 the ſecond Period, double the Quote 2, which is 4: ſet 4 under 7, the firſt Figure of the next Period.

Then demand how often 4 may be had in 47 right over it, and the anſwer is 9; ſet 9 in the Quote, and after 4, then multiply 49 by 9, the Figure laſt ſet in Quote, and it makes 441, which you muſt ſubſtract from 471 right over it, and there remains 30; cancelling 471, ſet 30 the remainder over it, juſt as in Diviſion.

```
001
4;810
87135 (295
4 the Square of 2.
 49 | double 2 more 9.
441 | 49 multiply by 9.
 585 double 29 more 5.
2925.585 multiplied by 5.
```

3. Draw

3. Draw a Line again under 441, as far as 5 the laſt Figure of the third Period, and double the Quote 29 ſet under it, *viz.* the 8 of 58, the double of 29 right under 3, the firſt Figure of the laſt Period, and the 5 backwarder.

Then demand how often 58 may be had in 313, the Figures over it, and the anſwer is 5, as in Diviſion. Set 5 in the Quote, and alſo after 58, which makes 585; then this multiplied by 5, the Figure ſet in the Quote, makes 2925, which taken from 3035, reſts 110 for a remainder, which proves it to be a Surd-Number, which will not be ſquared; for if 295 be ſquared, it is 87025, to which adding 110, the remainder makes juſt 87135, the firſt Number. To find the value of which Remainder, double the Root found, adding 1 to it, it is 591 for a Denominator, and 110 is the Numerator; as thus 295, and $\frac{110}{591}$.

Or to be more exact, Double the Remainder for a Numerator, and quadruple the Square-Root, adding 1 to it for a Denominator: thus 110 doubled is 220, and 4 times 295 with 1 added, is 1181, thus the Square-Root is 295 and $\frac{220}{1181}$.

Or elſe to gain a Decimal Fraction, add 2, 4, 6, 8 or 10 Cyphers to the Number propounded, and continue on the extraction to the end, then ſhall you have the Root and Decimal Fraction, as in the following Example is manifeſt.

Example the fourth, to find the Square-Root of 72 nearly.

Set down 72, and 10 Cyphers after it, point it, and quote it, and the operation will be as followeth, by the former rules.

Note, that ſo many Figures as you intend to have in the Decimal Fraction, twice ſo many Cyphers add to the Number, whoſe Square-Root is to be found.

Thus

Thus much for the Square-Root by the Pen, which is done by inspection, by the double and broken Line of Numbers; for right against 72, on the first part of the double Line is 8.485 on the broken Line of Numbers, to the 100 part of an Integer.

```
        00
       011|324
     1087761|39
    0844964479|196
   720000000000 (8.48498
```

Square of 8	64
Twice 8 and 4 added	164
164 multiplied by 4	656
Twice 84 and 8 added	1688
1688 multiplied by 8	13504
Twice 848 more 4	17964
17964 multiplied by 4	71856
Twice 8484 more 9	179689
179689 multiplied by 9	1618201
Twice 84849 more 8	1796988
1796988 multiplied by 8	14375904

CHAP.

CHAP. VII.

To find the Square-Root by the Lines of Numbers.

THe eaſieſt way is by the double, and broken Line of Numbers, in the print thus: When the Number of Figures in the Number propounded is even, *viz.* 2, 4, 6, 8 or 10, ſeek the Number in the firſt part of the double Line, and juſt againſt it in the broken Line is the Square-Root required; but when the Number of Figures is odd, ſeek it in the ſecond part of the Line to two Radiuſſes.

Example. The Square-Root of 5263 is 72.55 ſquared, or multiplied by it ſelf, is 5263.5025, being near to 5263.

Again, In 891 the Figures being odd, find it in the ſecond part of the two Radiuſſes, and right againſt it in the broken Line is 29. 86, the near Square-Root required.

But by the Line to 8 Radiuſſes, a mean Proportion between 1 and 891, gives 29.86, the Square-Root required.

So alſo, the middle between 1 and 5263 is 72.55, found as before.

But to do it by the Lines of Numbers on the Gauging-Rod, when the Figures be even, the middle between 100 at the end, and the Number propounded is the Square-Root.

But when the Figures be odd, then the middle between 10 in the middle and the Number propounded, counting toward 100, is the Square-Root required.

Thus the middle between 10 and 891, counted near 100, is 29.86 as before. Note always, the number of

of places in the Root as to Integers, is as many as the Number to be ſquared admits of Points, pointing the laſt, and every other backward thus 891 of 2 Figures, *viz.* 29, becauſe it admits of 2 Points.

The Square-Root of a Decimal Fraction is as ſurely and eaſily found as a whole Number.

For the middle between 1 and any Number leſs than 1, is the Square-Root of that Decimal Fraction.

Example. The Square-Root of 000729 is 008.56, the Square-Root of that Decimal Fraction required.

This is moſt certainly done by the Line to 8 Radiuſſes, as to Number of Places and Cyphers prefixed.

By the Lines one over another, when the Figures be even, the right hand 1 is Unite; when the Figures be odd, the left hand 1 is Unite.

CHAP. VIII.

To extract the Cube-Root of a Number by the Pen.

1. THe extraction of the Cube-Root of a Number, is to find out another Number, which being ſquared, (*viz.* muliplied by it ſelf) and then that Square cubed, *viz.* multiplied again by the Product of the firſt Multiplication, produceth the Number firſt given.

Thus 125 being a Number given, 5 his Cube-Root ſquared is 25, and 5 cubed is five times 25, *viz.* 125 the Number given.

In like manner to find the Cube-Root of 8, the Root is 2, for two times 2 is 4, and two times 4 is 8.

2. All Cube-Numbers are ſingle or compound. Single Cube-Numbers are only theſe 9 in the following Tablet, whoſe Cube-Roots are expreſt by 1 Figure only.

All

All other Cube-Numbers above 1000, are compound Cube-numbers, whose Cube-Roots are expreſt by more than 1 Figure, and theſe 9 ſingle Numbers are to be learn'd by heart, as the Multiplication Table is.

Root.	*Squar.*	*Cubes.*
1	1	1
2	4	8
3	9	27
4	16	64
5	25	125
6	36	216
7	49	343
8	64	512
9	81	729
10	100	1000

3. All Numbers are right ſingle, or compound Cube-Numbers, whoſe Roots cubed are juſt equal to the firſt given Numbers.

Or elſe they be Surd-Numbers, whoſe neareſt Roots cubed will not be equal to the firſt Numbers given, as 130, his neareſt Cube-Root is 5, which cubed is but 125, and 6 cubed is 216.

4. When you would find the Cube-Root of a Number by the Pen, you muſt prepare it with Points, to diſtinguiſh it into ſeveral Cubes or Periods, by putting a Point in the firſt place of Units, the fourth place of Thouſands, the ſeventh place of Millions, the tenth place of Thouſands of Millions, the thirteenth place to Millons of Millions, &c.

And note, that ſo many Points as the Cube-Number hath, ſo many Figures will be in the Cube-Root thereof.

The operation whereof mind, in theſe two Examples following, the Precept being very difficult to expreſs in words, and more hard to underſtand.

Example firſt, to find the Cube-Root of 54872.

Firſt ſet down the Number, point it, and make a Quote Line, as in the Square-Root, and as in the Example you ſee, the Number parted in two Periods, *viz.* 54 and 872, and the opperation for the firſt Period is thus.

1. Demand what is the greateſt Cube-Number in 54, and the little Table ſaith 27, whoſe Cube-Root is 3, ſet 3 in the Quote, and the Cube of 3, *viz.* 27, under 54, and ſubſtracting 27 from 54, put the remainder over as in Diviſion.

```
00
2700 )
54872 (38
 27   the Cube-Root of 3.
 27   the triple Square of 3.
  9   the triple of 3.
 279  the Sum, a Diviſor.
 216  the triple Squ. of 3 mult. by 8.
 576  triple of three 3, 64 times.
 512  the Cube of 8.
27872 the Sum.
```

So much for the firſt Period.

2. For the ſecond Period and remainder of the firſt. On a waſte piece of Paper, ſquare the Figure in the Quote, and then triple that ſquare, and drawing a Line, ſet it under 78.

Thus, three times 3 is 9, and three times 9 is 27, ſet 27 under 78; then triple 3 the Figure in the Quote, and it makes 9; ſet 9 under 7, the ſecond Figure of the next Period, and add them together, and ſet them under a new Line drawn, as in the work you ſee.

This Sum added, call the Diviſor.

Then demand how often 279 may be had in 2787? and you ſhall find 8 times; ſet 8 in the Quote, and draw a Line under 279 the Diviſor.

Then on a waſt Paper multiply the triple Square of 3, the firſt Figure in the Quote, *viz.* 27, by 8 the Figure laſt ſet in the Quote, and it is 216, ſet 216 down with the 6 under 8, the firſt Figure of the ſecond Period and the 21 backwarder.

Alſo, multiply the triple of the firſt Figure in the Quote, *viz.* 3. by the Square of 8 the ſecond Figure, *viz.* 64 by 9, and it is 576; ſet 576 a place forwarder, *viz.* the 6 right under 7 of the ſecond Period:

Alſo,

Alſo, ſet the Cube of 8, *viz.* 512, one place forwarder under 576, *viz.* 2 in 512 under 2 of the ſecond Period.

Then draw a Line, and add them together in one Sum, *viz.* 216, 576, and 512, and it makes 27872, which taken from 27 the remainder of the firſt Period, and 872 the ſecond Period, remains 0, becauſe it is a right compound Cube-Number, for 38 cubed is juſt 54872.

Example the ſecond.

Let the Cube-Root of 535387328 be required, conſiſting of 3 Periods.

1. Set down the Number, point it, and draw a Quote-Line, as in the work you ſee is done.

Then for the firſt Figure, demand what is the greateſt Cube-Number in 535, and the little Tablet ſaith 512, whoſe Cube-Root is 8; ſet 8 in the Quote, and the Cube thereof 512 under 535, and ſubſtract as in Diviſion, cancelling 535, and ſetting the remains 23 over 35, as in Diviſion.

0
03000
02384000
~~33~~38~~7~~328 (812
512 the Cube of 8.

192 the triple Square of 8.
024 the triple of 8.

1944 the Sum for a Diviſor.

192 the triple Square of 8 multiplied by 1.
24 the triple of 8 multiplied by the Square of 1.
1 the Cube of 1.

19441 the Sum added to ſubſtract.

19683 the triple Square of 81.
243 the triple of 81.

197073 the Sum added for a Diviſor.

39366 the triple Square of 81 multiplied by 2.
972 the triple of 81 mult. by the Square of 2.
8 the Cube of 2.

3946328 the Sum added to ſubſtract.

2. Then to find the next Diviſor, draw a Line under 512, and on a waſte Paper ſquare 8 the Figure in the Quote, and triple it, and ſet the triple ſquare under the Line, ſo as the Units in this triple ſquare may ſtand under the firſt Figure of the ſecond Period, as 192 under 233.

Set alſo the triple of 8, *viz.* 24 under 192, the triple Square of 8, one place forwarder, draw a Line, and add them together in one Sum, which Sum call the new Diviſor.

3. Then demand how often 1944 the new Diviſor may be had in 2338, the Figures right over it being the remainder of the firſt Period, and 2 Figures of the ſecond

cond Period, just as in Division, and the answer is 1; set 1 in the Quote for the second Figure of the Cube-Root.

Then on waste Paper multiply the triple Square of 8, *viz.* 192 by 1, which is but 192, set it directly under 192.

Also, Multiply 24, the triple of 8, by the Square of 1, which is but 1, and the Sum is but 24, set 24 under 24.

Also, Add in a place forwarder, right under 7, the last Figure of the second Period, the Cube of 1, which also is 1.

Draw a Line and add them together, and substract it from the Figures right over them, *viz.* 19441 from 23387, and the remainder is 03946.

Note, That in this Example, the adding of 1, the Cube of 1 to the Divisor, would have been the same, because 1 neither multiplies or divides, but for methods sake it is worded at large, which work may be saved when 1 comes to be a Figure in the Root.

4. Then for the next Divisor and Period, draw a Line under 19541, and on a waste Paper square 81, the two Figures in the Quote, which is 6561; then triple this square, which is 19683, set the last Figure 3 of this triple square right under 3 the first Figure of the third Period, and the other Figures backward toward the left hand.

Also, Set the triple of 81, which is 243, under 19683, one place forwarder, so as 3 the last Figure of 243 may be right under 2 the middle Figure of the third Period, and the 24 backwarder.

Then draw a Line, and add them together, and the Sum is the new Divisor, *viz.* 197073.

Then demand how often this Divisor may be had in 384632, the Number right over it being remainder of the second Period, and 2 Figures of the third, and the answer is 2; set 2 in the Quotient for the third Figure of the Root, then draw a Line, and multiply the triple

ſquare of 81, formerly ſet down by 2, and ſet it right under the triple ſquare, and it is 39366.

Multiply alſo the triple of 81, formerly ſet down by the ſquare of 2, laſt ſet in the Quote, which is 4, and it comes to 972, and ſet this right under the 243 the triple of 81.

Alſo, In a place forwarder, right under 8, the laſt Figure of the third Period, ſet down the Cube of 2, the Figure laſt ſet in the Quote, under 972, *viz.* 8.

Then draw a Line, and add them together, and the Sum is 39463288, to be ſubſtracted from 39463288 right over it, being the remainder of the firſt and ſecond Period, and all the Figures in the third Period, and the remainder is 00, therefore it is a right compound Cube-Number:

For if you ſhall cube 812, it will come to 535387328, the Number firſt propounded, to find the Cube-Root of it.

This work of the fourth Section is to be repeated as often as there be Periods in the Number propounded.

Alſo, In demanding how often the Diviſor may be had in the Number over it, beware you take not too much, for then you cannot make ſubſtraction, nor too little, for then the remainder is too much.

Alſo note, That when you meet with a Surd-Number, which will not be cubed, but have ſome remaining Fraction to bring out a Decimal Fraction, add 3, 6, 9, or 12 Cyphers to the Number propounded, and continue on the extraction to all thoſe Periods of Cyphers: and ſo many Periods of Cyphers as you have, ſo many Figures will be in the Decimal Fraction.

As for the extraction of Fractions of Numbers, the beſt way is to reduce it to a Decimal Fraction, and then it is done as a whole Number, of which you may ſee more in Mr. *Kerſies* Arithmetick, *Chap.* 33.

But for the Square-Root, or Cube-Root of all Decimal Fractions, the printed Paper in the Book will do it by inſpection, near enough for any Gaugers uſe, for this,

this difficult work by the Pen, is as easie as any other by the Lines single and triple, or more plainly by that to 8 Radiusses, as you find afterwards

An Example of a Surd-Number.

| 60
0 | 3932
24 00 00 | 576
37.24 (
27 the Cube of 3.
27 the triple Square of 3.
9 the triple of 3.
279 the Sum added for a Divisor.
189 the triple Square of 3 multiplied by 7.
441 the triple of 3 multiplied by the Square of 7.
343 the Cube of 7.
23653 the Sum added to substract from them over.
1107 the triple Square of 37.
111 the triple of 37.
11181 the Sum added for a Divisor.
2214 the triple Square of 37 multiplied by 2.
444 the triple of 37 multiplied by the Square of 2.
8 the Cube of 2.
225848 the Sum added to substract.
414852 the triple Square of 372.
1116 the triple of 372.
4149636 the Sum added for a Divisor.
1659408 the triple Square of 372 multiplied by 4.
17856 the triple of 372 mult. by the Square of 4.
64 the Cube of 4.
166119424 the Sum added to substract.

CHAP.

CHAP. IX.

To find the Cube-Root by Numbers.

THe eaſieſt way is by the triple, and ſingle Lines of Numbers on the print, in this manner.

1. When the Number whoſe Cube-Root is required, conſiſts of 1, 4, or 7 places, ſeek it in the firſt of the 3 Radiuſſes.

2. When it conſiſteth of 2, 5, or 8 places, ſeek it in the ſecond Radius.

3. When it conſiſteth of 3, 6, or 9 places, ſeek it in the third Radius, and note, the Line is ſo figured, with Units in the firſt Radius, Tens in the ſecond Radius, and Hundreds in the third Radius.

1	2	3
4	5	6
7	8	9

Thus the Cube-Root of 8 is 2.
The Cube-Root of 27 is 3.
The Cube-Root of 216 is 6.
The Cube-Root of 1728 is 12.
The Cube-Root of 17280 is 26.
The Cube-Root of 172800 is 56.
The Cube-Root of 8490200 is 204.

Wherein you may note, that the Root doth contain as many Figures as the Cube-Number admits of Points when the laſt and every third from the laſt Figure is pointed, as in theſe Examples you ſee.

By the Line to 8 Radiuſſes, always the Rule is thus,

The

The first of two mean Proportions, between 1 and the Number propounded, is always the Cube-Root required, to be found. Thus, if you divide the space on the Line of Numbers between 1 and 1728 into 3 parts, the first part counting from 1, will be at 12, the Cube-Root of 1728.

But by the Line of 2 Radiusses put on the Gauging-Rod, you must observe these Cautions.

First, The Number being written down and pointed, with a Point under the last Figure toward the right hand, and from thence at the fourth, the seventh, and tenth (if you have so many) toward the left hand, and so many Figures will be as Integers in the Cube-Root, as the Cube-Number hath Points.

Then when the Point falls on the last Figure toward the left hand, the middle 10 called 1 is the Unite, and the first of two mean Proportions forward from 10, between 10 and the Number propounded, is the Cube-Root required, found by dividing the space on the Line of Numbers, between 10 and the Cube-Number into three equal parts, and the first from 10, is the Cube-Root required.

But when the Point falls on the last but 1, then 1 at the beginning end is the Unite, and the Cube-Number will fall beyond 10 in the second Radius, and the Cube-Root in the first Radius.

But when it falls on the last but two, toward the left hand, then 100 at the end must be the Unite, and the Root and Cube falls backward between 100 and 10.

Thus the Cube-Root of 3136 is 14 $\frac{7}{19}$, being the first of three parts between 10 and 3136 in the second Radius.

The Cube-Root of 85184 is 44 found at the first of three equal parts, between 1 at the beginning and 85184 in the second Radius.

The Cube-Root of 195112 is 58, being the first of three equal parts between 100 at the end, and

195112

195112 in the second part counting from 100.

Thus this difficult Problem of the finding the Square and Cube-Roots of a Number by Arithmetick, by help of the Lines, is made very easie, yea, more than many other.

CHAP. X.

Of the Rule of Three Direct.

1. IN the Rule of Three is always three terms propounded, *viz.* 2 of Supposition, and 1 of Demand.

2. Two of the three terms are of one Denomination, (or to be made so,) and one of another Denomination.

3. Of the three terms given, that of Demand must be the third term, and the term of Supposition of the same Denomination with that of Demand, is always the first term in the Question; then the remaining term must needs be the second term in the Question.

4. Having found which be the first, second, and third terms in direct Proportion, the Rule is always thus:

As the first term to the second, so is the third to the fourth term required to be found, and by the Pen the Rule is.

Multiply the second and third terms one by another, and divide the Product by the first term, and the Quotient shall be the fourth term required.

Example.

If 12 Pound of any Commodity cost 3 Shillings, what cost 7 Pound? The answer is 1 Shilling 9 Pence;

for

for 7 multiplied by 3 produceth 21, and 21 divided by 12, quotes 1 Shilling and $\frac{9}{12}$ of a Shilliug, or 9 Pence.

By the Line of Numbers, the extent of a pair of Compasses on the Line of Numbers from 12 to 3, shall reach the same way from 7 to 1.75 (or 3 quarters, which is 9 Pence.)

Again, If 36 Gallons cost 10 Shillings, what cost 520 Gallons? The answer is 144 Shillings 6 Pence.

For 520 multiplied by 10, is 5200; and 5200 divided by 36, quotes $144\frac{1}{2}$ very near, or $\frac{16}{36}$ of 1 Shilling.

Or, the extent from 36 to 10, shall reach the same way from 520 to 144, and near a half the answer.

CHAP. XI.

Of the Line of Money.

THis Line of Money, and the Line of Numbers together, gives the Decimal Fraction of any Sum under 2 Shillings, or reduceth any Decimal Fraction to the Vulgar, 1 Pound, or 20 Shillings being the Integer, by inspection only.

Example. By the Line to 8 Radiusses, where it is best and plainest seen.

First, 1 Pound being called 1, is just at 1 in the middle of the Lines when they are put together.

Then note, If 20 Shillings is but 1, any Number or Portion of Money less than 20 Shillings is less than 1.

Therefore, the Decimal for 18 Shillings is 09, (but in some Books noted thus, .9) 10 Shillings is in Decimals 05, 8 Shillings is 04, 2 Shillings is 01, being equal to 24 Pence, which on the Joynt-Gauging-Rules is at 100 at the upper end near the Joynt; then that being 01, the middle 10 is 001, and 1 at the beginning 0001.

Then.

Then in a Radius leſs, 18 Pence is 0075, 12 Pence is 005, 6 Pence is 0025, 1 Peny is 000416 in a Radius leſs, 1 Farthing is 000104 near 1 at the beginning of the Numbers on Four-foot-Gauging-Rods.

So that by the Line to 8 Radiuſſes, you have the true value of the Decimal Fraction, whoſe uſe ſhall be partly ſeen in theſe Examples.

1. *If* 100 *coſt* 5 *Pound, what coſt* 1 ?

Juſt againſt 50 on the Numbers on Gauging-Rules, is on the Money-Line 12 Pence, the exact price, when 100 coſt 5 Pound.

2. *If* 100 *coſt* 2 *Pound* 4 *Shillings, what coſt* 1 ?

Juſt againſt 22 on the Numbers on the Line of Pence, is 5 Pence 1 Farthing, and $\frac{1}{10}$ of a Farthing.

3. *If* 100 *coſt* 4 *Shillings, what coſt* 1 ?

Juſt againſt 2 on the firſt Radius, which is 4 Shillings when 10 in the middle is 1 Pound, on the Money-Line is 1 Farthing, and $\frac{9}{10}$ of a Farthing, the exact price of 1.

4. *For any Price* per 100 ?

Under 2 Shillings ſeek it in the Line of Pence, and that is the exact anſwer.

Example. At 9 Pence *per* 100, let a Penny be parted into 100 parts, 9 of thoſe parts is the exact Anſwer.

Then by Reduction, If 100 parts be 4 Farthings, what is 9 parts ? The anſwer is $\frac{1}{25}$ or 36 parts of a Farthing in 100 parrs.

5. *If* 100 *coſt above* 10 *Pound* per 100, *count thus*, Example, *At* 65 *Pound* 15 *Shillings* per 100, *what coſt* 1 ?

Count 65 Pound 15 Shillings at 65.75, then obſerve 6 $\frac{1}{2}$ doubled, is 13 Shillings; then ſeek the 075 over, on the firſt Radius before 10 in the middle, and juſt againſt it in the Line of Pence is 7 Farthings $\frac{1}{5}$, in all 13 Shillings 1 Penny 3 Farthings $\frac{1}{5}$, the exact price of 1, when 100 coſt 65 Pound 15 Shillings 0 Pence.

And

And on the contrary, having the price of 1, to find the price of 100.

For any price for 1, above 1 Farthing, and under 2 Shillings, ſeek the price of 1 on the Line of Pence, and juſt againſt it on the Numbers is the price of the 100.

Example. At 2 Farthings for one, is 4 Shillings, 075 for a 100, then the 075 count in the firſt Radius, and in the Pence, is 7 Farthings $\frac{1}{5}$, in all 4 Shillings 1 Penny 3 Farthings $\frac{1}{5}$, the exact anſwer.

At 7 Pence for one, is 2 Pound 18 Shillings, 166: which ſought on the Line of Numbers, the Line of Pence ſheweth juſt 4 Pence over, in all 2 Pound 18 Shillings 4 Pence, the price of 100, when one coſt 7 Pence.

Again, At 18 *Shillings* 7 *Pence* 2 *Farthings* per 1, *what coſt* 100?

Note, 18 Shillings is at 9, on the firſt part of the Line of Numbers, and here 9 ſtands for 90 Pounds, then ſeek the odd 7 Pence 2 Farthings on the Line of Pence, and right againſt it on Numbers is 3.125; the 3 is 3 Pound, and the 125 is 2 Shillings 6 Pence more, in all 93 Pounds 2 Shillings 6 Pence, the exact anſwer.

Again, At 8 *Pound* 5 *Shillings* per 1, *what coſt* 100?

Firſt, To the 8 Pound add 2 Cyphers, and it becomes 800 Pound: then to the 5 Shillings add 2 Cyphers, and it is 500 Shillings, or 25 Pound, in all 825 Pounds, the price of 100 when 1 coſt 8 Pounds 5 Shillings.

The uſe of the Line of Pence in any ordinary Queſtion, in many Examples.

1. *If two Gallons and* $\frac{1}{2}$ *coſt* 7 *Pence* 3 *Farthings* $\frac{3}{4}$ *of a Farthing, what coſt* 36 *Gallons.*

The extent from 2 $\frac{1}{2}$ on Numbers to 7 *per* 3 Farthings $\frac{3}{4}$

$\frac{3}{4}$ on the Line of Pence, ſhall reach from 36 on Numbers to 475, which is 9 Shillings 6 Pence; for 4 doubled is 8 Shillings, and 5 more is 1 Shilling, and 25 Farthings more is 6 Pence, in all 9 Shillings 6 Pence.

2. *If three Pound three quarters of any Commodity coſt 7 Pence 3 Farthings and $\frac{1}{2}$ Farthing, what coſt 38 $\frac{3}{4}$?*

The extent from 3 $\frac{3}{4}$ counted on the Line of Numbers to 7 Pence 3 Farthins $\frac{1}{2}$ Farthing counted on the Line of Pence, ſhall reach the ſame way from 38 $\frac{1}{4}$ counted on Numbers, to 3.40 on the Line of Numbers, which being reduced is 6 Shillings 9 Pence 2 Farthings and $\frac{1}{2}$, the anſwer.

3. *If 112 Pound coſt 28 Shillings, what coſt 1 Pound?*

The extent from 112 to 1 Pound 8 Shillings counted on the Line of Numbers at 14, being laid the ſame way from 1 on the Numbers, on the Line of Pence, gives 3 Pence the price of 1 Pound.

4. *If 1 Pound coſt 5 Pence 2 Farthings, what coſt 112 Pound?*

The extent from 1 on the Numbers, to 5 Pence 2 Farthings on the Line of Pence, being laid from 112 on the Numbers, gives 2 Pound 11 Shillings 4 Pence.

5. *If 112 Pound coſt 2 Pound 10 Shillings 6 Pence, what coſt 21 C. 3 q. 12 l.?*

The extent from 112 to 2.525 on the Line of Numbers being laid the ſame way from 2448 the Pounds in 21 C 3 q. 12 Pound, ſhall reach to 55 Pound 2 Shillings on the Line of Numbers.

By the Pen work thus.

Multiply 2.525, the Decimal of 2 Pound 10 Shillings 6 Pence by 2448, the Pounds in 21 C. 3 q. 12 Pound, and the Product is 6181.200: which being divided by

112,

112, quotes 55.108, or 55 Pounds 2 Shillings 2 Pence.

Or the extent from 1, for 1 C. weight to 2.525, the price of 1 C, ſhall reach the ſame way from 21 C. 86. (the Decimal for 3 q. and 12 Pound) to 55 Pound 2 Shillings 2 Pence.

6. If 6 Ounces $\frac{1}{2}$ coſt 20 Pence 3 Farthings, what coſt 1 C. 1 q. and 15 Pound?

The anſwer is 39 Pounds 2 Shillings 10 Pence, being beſt wrought thus, at two operations.

The extent from 5 $\frac{1}{2}$ on Numbers, to 20 Pence 3 Farthings on the Line of Pence, being laid the ſame way from 16, the Ounces in 1 Pound to 2.518, being 5 Shillings 0 Pence near 2 Farthings.

Then the extent from 1 on Numbers to 2.518, being laid the ſame way from 155, the Pounds in 1 C. 1 q. and 15, ſhall reach to 39 Pounds 2 Shillings 10 Pence, the anſwer.

To work this by the Pen, find the price of 1 Ounce exactly, and multiply it by the Ounces in 1 C. 1 q. and 15 Pound, and the Product is the anſwer.

So much at this time for Numbers and Pence.

CHAP. XII.

Of ſimple Intereſt.

1. TO find the Intereſt of any Sum for 1 Year.

The extent from 100 to the rate allowed for 100 Pound in one Year, ſhall reach from any other Sum to its proportional Intereſt.

Example. The extent from 100 to 6 Pound, the Intereſt due for 100 Pound in one Year, ſhall reach

from 75 Pounds to 4 Pounds 10 Shillings, the Interest due for 75 Pounds at one Years end.

By the Pen, say by the Rule of Three, As 100 to 6, so is 75 to 4-5, or 4 Pounds 10 Shillings, thus: 6 multiplied by 75, the Product is 450, and the dividing by 100, is done by cutting off two Figures.

2. To find the Interest of any Sum for any time.

The extent from 100 to the time given (being counted as in the Line to 8 Radiusses,) shall reach the same way from the Money propounded to the Interest due for that Sum at the time propounded.

Example. What is the Interest of 125 Pound in one Year and 3 Months, or 91 Days?

The extent from 100 to 0.74, the place for 1 Year and 3 Months, being the half of 15, the Months in 1 Year and quarter, shall reach the same way from 125 Pound to 9 Pound 7 Shillings 5 Pence, or 9.370 (which is reduced thus.)

Every Unite in the first place of Fractions is 2 Shillings, 5 in the second place is 1 Shilling the Remainder above 5, or those under 5 are tens of Farthings, the third place is Farthings, thus 370 is 7 Shillings 5 Pence.

By the Pen, the best way is to bring the time to Days, and the Money to a Decimal Fraction. Then thus,

The Interest of 1 Pound for 1 Day, is .000164384 found, by dividing 6 by 365; and by this Table by Addition or Multiplication, the Interest of 1 Pound, any number of days may be had.

Then multiply that Sum by the Money, and the Product is the answer required.

Thus .000164384 multiplied by 456, the Days in 1 Year 3 Months, the Product is 074959104; then this Sum multiplied by 125 the Sum

1	. 000164384
2	. 000328768
3	. 000493152
4	. 000657536
5	. 000821920
6	. 000986304
7	. 001150688
8	. 001315072
9	. 001479456
10	. 001643840

Sum of Money, the Product is 9.369888000, which reduced, is 9 Pound 7 Shillings and near 20 Farthings, or 5 Pence; for rebate or discount by the Pen thus: find the simple Interest of 1 Pound for the time propounded, as before by Multiplication, thereunto add a Unite, then by it divide any other Sum, the Quote is the presenr worth by the Line thus.

The extent from 100 to the Months, (counting 106 one Year, 112 two Years, 118 three Years) shall reach from the Sum of Money to the increase of Principal and Interest.

Or, Being laid the contrary way to the present worth, discounting for the time.

Example. What is the increase of 125 Pounds in 40 Months?

The extent from 100 to 120 being laid the same way from 125, gives 150 the Principal and in Increase in 40 Months, but laid the other way, gives 104 Pounds 4 Shillings the present worth.

Before I come to the use of the Rule, in gauging of close or open Casks, it is necessary to shew how to measure any plain flat Measure, or solid Measure, as followeth by their Names and Figures, both as superficial and solid Measure, both by the Pen and Line of Numbers.

PROB. I.

To measure a Geometrical Square, and a Cube.

A Geometrical-Square is a Superficial-Figure, made up of four equal Sides, and four equal Angles, as *a*, *b*, *c*, *d*, having the sides *ab*, *bd*, *cd*, and *ac* equal, and the Angles at *a*, *b*, *c*, *d* equal also. And a Cube is a Solid, erected upon such a Plain, as this whole Figure represents, every way six Inches, and measured by the Pen thus:

Multiply the length by the bredth, and that Product is the Superficial-content, then multiply this Product or Superficial-content ; by the depth, and that laſt Product is the Solid-content.

Example. In this Figure of 6 Inches ſquare, ſix times 6 is 36, the Product of 6 multiplied by 6 being the Superficial-content of ſuch a Square-Figure. Note alſo, that any Number multiplied by it ſelf, is often called Squaring.

Then this Product 36 multiplied by 6, produceth 216 the Solid-content of ſuch a Cube 6 Inches every way ; and this ſecond multiplying a Number, is called a cubing a Number, and frequently uſed in the following Diſcourſe.

Thus: 216 Inches is the Solid-content of a Cube 6 Inches every way.

By the Line of Numbers this is thus performed :

The extent from 1 to 6 reaches at once repeating or turning of the Compaſſes from 6 to 36, and at two repeatings or turning the Compaſſes to 216, the Cube-Number required.

PROB. II.

To meaſure a Parallelogram, or a Parallelopipedon, by the Pen or Numbers.

A Parallelogram is a long or plain Square : a Parallelopipedon is a ſolid-figure erected upon that Plain.

Thus, *a*, *b*, *c*, *d* of the ſecond Figure is a Plain or Parallelogram of 18 Inches long, and 4 Inches broad ; then four times 18 is 72, the Superficial-content of the Plain or Parallelogram, and called the Rectangle. Then again ; the Piece being 6 Inches thick or high, the Rectangle or Product 72 being multiplied by 6, gives 432 for the ſecond Product, content, or ſolidity of the Paralelopipedon.

By

By the Numbers thus.

The extent from 1 to 18 being laid the same way from 4, the bredth gives 72 the Product or Superficial-content of the Parrallelogram.

Then secondly, The extent from 1 to 72, reaches the same way from 6 the depth to 432 the Solid-content of the Parallelopipedon or Solid-figure.

This Rule serves to measure Timber, or Stone, or like Square-solids. Also note, that if you find the side of a Square equal to 4 and 18, then twice turning the Compasses serves as well, the Square equal being a Geometrical mean Proportion, or the middle between 4 and 18 on Numbers being 8 and $\frac{1}{2}$; then the extent from 1 to 8.48 the middle between 4 and 18 being twice repeated from 6, gives 432 as before.

PROB III.

To measure a Rombus, or a Romboides.

A Rombus is a Geometrical Square bent awry, as a Pane or long Quarry of Glass; and a Romboides is a long Square or Parallelogram, out of Square at the ends, as the Figures following shew, & measured thus.

Multiply one side as *a*, *b* in the Rombus, or as in the Romboides by *b*, *e*, or *c*, *e* the neareſt distance between the sides, and that Product shall be the content of the Rombus or Romboides.

Example. There is a Rombus whose 4 sides are 6 Inches each, but the Angles or Corners at *a*, *b*, *c*, *d* are not equal, but *a* and *d* are acute or sharp Angles, and *b* and *c* are obtuse or blunt Angles; but the neareſt distance from *a*, *b* to *c*, *d*, is *b*, *e* 5 Inches and $\frac{1}{2}$, though *b*, *d* be 6 Inches; then I say, the Product of 5 $\frac{1}{2}$ multiplied by 6, is the Superficial-content of the Rombus, *viz.* 33; which in the Square of the same measure was 36: Also in the Romboides 13 the length of

of one ſide multiplied by 5 ½, gives 71 ½ the Superficial-content: but if a Solid ſhould be erected on this Baſe or Plain, then the depth thereof multiplied by the Superficial-content, ſhall give the Solid-content required. By the Line of Numbers, the extent from 1 to the length of one ſide 13, being laid the ſame way from 5 ½, the neareſt diſtance between the ſides gives 71.5, the content of the bottom or baſe.

Then if ſuch a Veſſel be 8 Inches deep, the extent from 1 to 71.5 ſhall reach from 8 to 572 the Solid-content of ſuch a Romboidal Figure.

PROB. IV.

To meaſure a Triangle.

A Triangle is a Figure contained by three ſides, either equal or unequal, which are right-angled, or obtuſe, or acute-angled, or equilateral of 3 ſides all equal, but all are meaſured by this general rule.

Multiply the Baſe or longeſt ſide by the Perpendicular, or neareſt diſtance from that ſide to the oppoſite (corner or) Angle, and half that Product ſhall be the Superficial-content of the Triangle.

Example. Let *a*, *b*, *c* be a Triangle to be meaſured, *a*, *b* the Baſe or longeſt ſide 24 Inches, and the Perpendicular or neareſt diſtance from the longeſt ſide to *c* the oppoſite corner is 6; then I ſay 6 multiplied by 24 hath 144 for the Product, whoſe half 72 is the true Area of the Triangle.

By the Numbers thus: The extent from 2 to 24 ſhall reach the ſame way from 6 to 72, the content without halving, becauſe you went from 2 inſtead of 1.

This is the plain and general way for all Triangles whatſoever.

But if you cannot come to meaſure the Perpendicular, then thus by the meaſure of the three ſides only.

I. Add

I. Add the 3 sides together to get the Sum, and half that for the half Sum of the sides, called *Perimeter* and *Semiperimeter.*

II. From the Semiperimeter substract each side severally to get the three differences.

III. Multiply one difference by another difference, and then this Product by the third Difference, and yet again this last Product by the Semiperimeter, after the manner of continual Multiplicarion.

IV. The Square-Root of this last Product, is the Superficial-content of the Triangle.

Example.

The three sides of the Triangle in this Example are 24, the longest and 13.416, the other two being both alike.

The Sum of the three sides is ——— 50.832
The half Sum or Semiperimeter is ——— 25.416
The three differences between the half Sum and each side severally are ——— 1.417
And ——— 12 000
12 000

Then the Product of 12 multiplied by 12, is 144, and the Product of 144 multiplied by 1.417, is 204.048 for a second Product.

Lastly, The Product of 204.048 being multiplied by 25.416, the half Sum, is 5185.288016, whose Square-Root is near 72, the Area found as before: for just against 5185 sought on the double Line of Numbers, on the broken Line is 72, the content Superficial, or Area, as before.

Then if this Triangle have depth, as here 12 Inches deep, then this Area of the Base or Bottom 72, being multiplied bo 12, the depth hath 864 for the Solid-content of such a Triangle Vessel.

But

But if any ſhall deſire to find what the Perpendicular will be in any Triangle by Arithmetich, the Rule is:

I. Square every ſide ſeverally (that is, multiply each ſide by it ſelf.)

II. Then add the Square of the Baſe, or longeſt ſide, to the Square of the ſhorteſt ſide.

III. And from the Sum ſubſtract the Square of the other ſide, halving the reſidue.

IV. Divide half the reſidue or remainder by the Baſe, and ſquare the Quotient.

V. Subſtract this ſquare from the ſquare of the leaſt ſide, firſt added to the Square of the Baſe, noting the remainder.

VI. The Square-Root of this Remainder, is the Perpendicular required.

Example. In the former Triangle 24 and 13.416 each.

The Square of 24 the longeſt ſide, is	576
The Square of 13.416, is near	180
The Square of the other ſide, is near	180

The Sum of the greateſt and leaſt Square added, is 756, from whence taking 180, remains 576, whoſe half is 288.

Then 288 divided by 24 the Baſe, hath 12 for a Quotient, the place on the Baſe where the Perpendicular will fall from the neareſt end.

Then 12 the Quotient ſquared, is 144, and this Square taken from the leaſt Square added, *viz.* 180, remains 36.

Laſtly, The Square-Root of 36, is 6, the true Perpendicular required.

Note, Theſe Squarings are done by the Line of Numbers as ſoon as written down, but the Addition and Subſtraction by the Pen, and the extracting of the Roots

Roots by a look of your Eye on the double and broken Line.

PROB. V.

To measure a Trapesia.

A Trapesia is a Figure of any four unequal sides, as this Figure sheweth, and the measuring of it is best done, by bringing it into two Triangles, by drawing of one Line from corner to corner, as in the Example, then the measure of those two Triangles is the true Area or Superficial-content of such a Figure; then that Area multiplied by the depth shall give the Solid-content.

Example. Let *a*, *b*, *c*, *d* be a Trapesia, and the Line *a*, *d* drawn from *a* to *d* be 24 Inches, and the neareſt distance from that Line to *c* one corner 6 Inches, and from *b* the other corner to that Line continued 12 Inches, I say the Product of 12 the half Diagonal Line, and 18 the Sum of the two Perpendiculars multiplied, is 216, the Superficial-content.

Or, The Sum of the two Triangles 9 multiplied by 16, and 6 multiplied by 12, is 144, and 72, whose Sum is 216 as before.

Or again, Thus: the half of 32 one side, and 19 the other side (whose Sum is 51) *viz.* 25.5 being multiplied by the half Sum of 17 the two neareſt distances from *c* to *a*, *b*, and from *d* to *a*, *b*, one being 9 Inches, the other 8 Inches, and the Sum 17, the half Sum 8.5, hath for a Product 216.75 near as before.

Then having the Area of the Base 216, multiply it by the depth in Inches, gives the content in solid Inches.

As, Suppose such a Cooler be 8 Inches deep, 8 times 216 is 1728 the solid Inches: then if you divide 1728 by 282, the Cube Inches in one Ale Gallon, you have 6 Gallons, and 36 Inches over, being a Pint.

By

By the Numbers, The extent from 8 to 8.5, being laid the ſame way from 25.5, gives 216.75 as before.

Or, The extent from 2 to 24, the length of the Diagonal-Line *a*, *d*, being laid the ſame way from 18, the Sum of 12 and 6 the two Perpendiculars, give 216¾ the Sum as before.

Or, As 1 to 12 the half of 24, ſo is 18 to 216; or as 1 to 9, the half of 18, ſo is 24 to 216.

PROB. VI.

To meaſure a many-ſided irregular Figure.

Theſe are to be reduced to Triangles, or Trapeſias, by drawing of Diagonal-Lines from one corner to another, and then meaſured and caſt up ſeverally by the two laſt Rules for a Triangle and a Trapeſia: then having found the Superficial-content, multiply it by the depth, and the Product is the Solid-content in like meaſures.

Example. Let *a*, *b*, *c*, *d*, *e*, *f* be a many-ſided Figure, as a Cooler, or ſuch like Veſſel to be meaſured.

Draw the Diagonal-Line *f*, *b*, and take the meaſure thereof exactly in Inches, and 10 Parts, which let be 31 Inches: Find alſo the exact meaſure from *a* to *b*, *f*, and from *e* to *b*, *f*, *viz.* 10.8 and 5.2, in all 16, the half of 16, *viz.* 8 multiplied by 31, gives 248 the Area of that Trapeſia.

Again, The Diagonal *d*, *b* meaſured, is 24, and the two Perpendiculars from *c* and *e* to the Diagonal *d*, *b*, is 12.7; then 12.7 multiplied by 12 the half of 24, hath for a Product 150.40 the Area of the other Trapeſia contained in this irregural many-ſided Figure; then 248 and 152.40, is 400.40 the true Superficial-content of the Figure *a*, *b*, *c*, *d*, *e*, *f*.

But if you have a Cooler in this form, then multiply 400.4 by the depth. Suppoſe 9 Inches, then 400.4 multiplied by 9, produceth 3603.6 Solid-Inches; then this

this 3603.6, divided by 282 the Cube-Inches in one Ale Gallon, quotes 12 Gallons 6 Pints $\frac{1}{4}$ near.

The operation by Numbers is as before, the extent from 2 to 31 reaches the same way from 16 to 248.

And the extent from 2 to 24 reaches the same way from 12.7 to 152.4: then the 2 Numbers put together, is 400.4: then the extent from 1 to 9 shall reach the same way from 400.4 to 3603.6: then the extent from 282 to 1 shall reach from 3603.6 to 12 Gallons, 82 parts, or 6 Pints.

But having gotten the Area of the Base, you may work thus for Gallons. The extent from 282 the Cube-Inches in 1 Ale-Gallon to 400.4 the Area of the Base or bottom in Inches, shall reach the same way from 9 the depth to 12 Gallons, 82 parts as before.

PROB. VII.

To measure a regular Poligon.

1. A regular Poligon is when the sides are more than 4, and yet all equal, as of 5, 6, 7, 8, 9 or 10 equal sides, as the Figure annexed is of 6 equal sides, and the plain way to measure it is thus:

Multiply the half-Sum of the measures of all the sides, by the measure of the Line drawn from the Center to the middle of any one side, and the Product shall be the Superficial-content.

Example. Six times 12 (the measure of one side) is 72, whose half 36 multiplied by 10.4, the measure from *c* to 12, gives 374 for the Superficial-content of this Hexagon or six-sided Figure. Or else most exactly by help of the following Table, as followeth in several Examples.

2. The Poligons side given, to find the Area. Square the side of the Poligon given, and multiply it by the Number in the first Column for that Poligon, and that

is the Area in Inches or like measure, thus: 12 squared is 144, and this multiplied by 259.808 (the Number in the first Column of the Table for an Hexagon,) produceth 374.12352, the Area of the Hexagon, whose side is 12, five Figures being cut off, *viz.* the 3 for Fractions, and 2 as divided by 100. By Numbers thus:

A Table of Areas the Sides being 10. *And a Table of Sides the Area being* 100.

Polig.	Area the Sides 10.	Sides the Area 100.
Perp.	8 666	13.158
Trig.	43.301.	15.197+
Squa.	100.000	10.000
Pent.	172.048+	7.624.
Hex.	259 808+	62.04+
Hept.	363.391	5.246
Octa.	482.843+	4.551
Non.	618.182.	4.022.
Dec.	769.421+	3.605.
Und.	936.566.	3.268+
Duo.	119.615.	2.989.
R. Cir.	314.159	5.462

A Table of the Sides, Areas, and Sides of the Squares equal, of the first 12 *Poligons, inscribed in a Circle, whose Diameter is* 20, *calculated by* E. H.

Sides	Areas.	Sid. of the Squar. eq.	Pol.
15 0000			P
17.32051	129 90381	11.39753	3
14.14214	200.00000	14.14213	4
11.75570	237.76413	15.41960	5
10.00000	259.80762	16.11855	6
8 67768	273.64112	16.54210	7
7.63267	282.84271	16 81793	8
6.84040	289.25442	17.00748	9
6.18034	293.89263	17.14330	10
5.63464	297.35186	17.24389	11
5.17638	300.00000	17.32051	12
10.00000	314.15926	17.72468	R

The extent on the Line of Numbers from 10 the Hexagons side in the Table, to 12 the Hexagons side given, being twice repeated, from 259.808, the Number in the Table for the Area of a Hexagon, whose side is 10, shall reach to 374.12352, the Area of the Hexagon whose side is 12.

3. On the contrary; having the Area, to find the side. Find the Square-Roots of the Area in the Table, and also of the Area given; then by the Rule of Three say, As the Square-Root of the Tabular Area to the Square-Root of the given Area, so is the side in the Table to the side required to be found.

Example.

Example. The Square-Root of 259.808, the Tabular Area is 16.12. The Square-Root of 374.12352, is 19.345; then as 16.12 to 19.345, so is 10 the side of the Poligon in the Table, to 12 the side of the Poligon required.

By the Line of Numbers thus: The half distance (or middle) between 259.808, the Area in the Table, and 374.12352 measured on a Line of Numbers, shall reach on the same Line from 10 to 12.

Then having the Area in Inches, multiply it by the depth, gives the solid Area in Cube-Inches, which you may reduce to Gallons or Barrels, by the proper Divisors, as in a Table following.

4. But if you seek the Areas or Sides of any of these Poligons, as inscribed in a Circle, or the sides of a Square equal unto them, then the second part of the Table will help you to them, in this brief manner, either by Pen, or Compasses and Numbers.

Having the side of a Poligon given, to find the circumscribing Circles Radius.

Add Cyphers to the given Side, and divide by the Number in the Table, the Quote is your desire.

Thus the Side given being 120 of a Pentagon, 7 Cyphers added, and divided by 11.7557, quotes 102.04, the Radius of a Circle to inclose a Pentagon, whose Side is 120.

Or, The extent from 11.7557, the Number in the Table, to 120, shall reach from 10 the fixed Radius in the Table, to 102.04, the Radius or Semidiameter required.

5. On the contrary: having the Radius, to find the Side.

The extent from ten the fixed Radius, to 102.04, the given Radius, shall reach the same way from 11.7557 the fixed Side, to 120 the Side required.

Or, Multiply the given Radius by the fixed Side, and the Product is the Side required.

6. Or, Having the Area of a Poligon given, to find his Side, Circles Radius, or Side of a Square equal.

Find the Square-Roots of the Tabular Area, and the given Area, as in the third Example. Then as one Square-Root is to the other, ſo is the Tabular Side to the enquired Side; or ſo is the Tabular Square equal to the enquired Square equal; or ſo is the Tabular Semidiameter 10 to the inquired Semidiameter.

By the Numbers. The half diſtance between the Areas, reaches from Side to Side, from Square to Square, and from Radius to Radius, required.

Alſo note, That by the double and ſingle, or broken Line, you may ſave the trouble of dividing the Space between the two Areas, thus:

Take the extent between 374.123 the given Area, and 259.808 the Tabular Area, on the double Line, and on the ſingle Line, or broken Line, it ſhall reach from 10 to 12 the Tabular Side, to the enquired Side, and from 16.118 the Tabular Side of the Square equal to 19.35, and from 10 the Tabular Semidiameter, to 12 the enquired Semidiameter.

PROB. VIII.

To meaſure a Pyramid.

But if on the Plains of the regular Poligons, ſhould Solids be erected with Perpendicular Sides.

Then multiply the Area of the Baſe by the Poligons altitude, and the Product ſhall be the Solid-content.

But if the Sides proceed Taper, and meet in a point like a ſpire Steeple, then multiply the Baſe by one third part of the Perpendicular altitude, and the Product is the Solid-content.

Example. The Area of the Baſe of the Hexagon laſt mentioned, is 374.123. Then ſuppoſe the Perpendicular altitude be 38, one third is 12.666, the Product after Multiplication is 4738.642+.

But

But if the Perpendicular height be not eaſily meaſured, you may find it by the ſlant height thus:

Square the half Baſe, *viz.* 10.4: Square alſo 40.15 the ſlant height, and take one Square from the other, and the Square-Root of the Remainder is the Perpendicular height.

Thus, The Square of 10.4 is 108.16, the Square of 39.48, the ſlant height, meaſured on the middle of one plain, is 1558.02: Then 108.16 taken from 1558.02, remains 1450, the Square of the Perpendicular height, whoſe Square-Root is 38 near.

This Rule will find the Perpendicular altitude in leaning Tuns, and may be of good uſe ſometimes.

Or to get the Solid-content in Wine, or Beer Gallons, or Barrels.

The extent from 282 to 374.123, ſhall reach from 12.666, one third of the altitude, to 16 Gallons 82 parts.

Or, The extent from 16.796 (the Gauge-Point for a Beer-Gallon in Square-Veſſels, being the Square-Root of 282) to 61.25, the ſide of a Square equal to 374.123, being twice repeated from 12.666, one third of the Perpendicular height gives 16 Gallons 82 parts as before.

By the Pen: Multiply 374.123, the Area of the Baſe, by 12.666, one third of the height, and the Product is 4738.6519- then this divided by 282, the Cube-Inches in a Gallon, quotes 16 Gallons 227 Inches.

Or, If you multiply it by 3547, the Arithmetical complement of 282, the Product is 16 Gallons and 82 parts as before.

PROB. IX.

To measure a Prism.

A Prism is a Figure like the roof of a House, with upright Rafters at each end, as a Pyramid had hip Rafters at each end, or sloping every way.

And to measure this, first find the Area of the Base, and multiply it by half the Perpendicular altitude, and the Product is the content.

Example. Let *a*, *c*, *d*, *f*, represent the Base of a Prism, and *a*, *b*, *c*, the end of it: the Area of the Base 20 by 30, is 600: then get the Perpendicular altitude by measuring, or by the last Rule in the Triangle, which you will find to be 2.28, multiplied by 600, gives 1368, the Cube-Inches in such a Figure.

Or by Numbers. The extent from 1 to 600, shall reach the same way from 2.28 to 1368, as before.

Or to bring out the Content in Beer Gallons, By the Pen.

Multiply the Area of the Base by half the Altitude, the Product is the Solid-content in Cube-Inches.

Then this Product divided by 282 the Square of the Gauge-Point (16.796, for Square Vessels) the Quotient is the Content in Ale Gallons.

Thus 600 the Base, multiplied by 2.28, the half-altitude of the Prism, produceth 1368.

Then 1368 divided by 282, the Quotient is 4 Beer Gallons and 240 Inches.

Or, By the Line of Numbers thus:

The extent from 16.796, the Gauge-Point for an Ale Gallon in Square-Vessels, to 24.5, the Side of the Square equal to 600 the Area, (being the middle between 600 and 1 on Numbers.)

That extent being laid twice the same way from 2.28, the half altitude of the Prism, gives 4 Gallons 86 parts, the Solid-content in Ale Gallons.

PROB.

PROB. X.

To measure a Circle, or an Elipsis, or Oval, and their Segments.

1. Having the Diameter, to find the Circumference.

Multiply the Diameter by 3.14159, the Circumference of a Circle whose Diameter is 1, and the Product is the Circumference

Thus 11.3 multiplied by 3.14159 hath 35.499967.

Or, The extent from 10 to 11.3, shall reach the same way from 3.14159 to 355.

2. The Circumference given, to find the Diameter.

Multiply the Circumference given by .318309, the Diameter of a Circle whose Circumference is 1, and the Product shall be the Diameter.

Thus 35.5 multiplied by .318309 a Decimal Fraction, hath 11.3009695, or 11 and 3 tenths for Diameter.

Or, The extent from .318309 to 1, shall reach from 35.5 the contrary way to 11.3 the Diameter.

But rather say in direct Proportion thus.

The extent from 3.14159 the Circumference, to 1 the Diameter, shall reach the same way from 35.5 the Circumference to 11.3 the Diameter.

3. The Diameter and Circumference given, to find the Area of a Circle.

Multiply half the Diameter by half the Circumference, and the Product shall be the Area, Thus: 5.65 the half of 11.5 the Diameter, multiplied by 17.75 the half of 35.5, produceth 100.2875.

Or, Multiply the Square of the Diameter by .078539, the Area of a Circle whose Diameter is 1, and the Product shall be the Area required.

Thus 144 the Square of 12 multiplied by .78539, hath 113.09616 the Area. By Numbers thus:

The extent from 1, a fixed Diameter, to 12 the given Diameter,

Diameter, ſhall reach from .78539, the fixed Area, to 113.09616, the enquired Area, at twice turning of the Compaſſes.

4. The Circumference given, to find the Area, or Content of a Circle.

Multiply the Square of the Circumference by .079578, the Circumference when the Area is 1, and the Product ſhall be the Area.

Thus the Square of 35.5, 1260.25, multiplied by .079578, produceth 100.28817650, the Area of a Circle, whoſe Circumference is 35.5.

By Numbers thus: The extent from 31.4159 to 355, being twice repeated from 78.539, the given Area, reaches to 100.288, the Area required.

5. The Diameter given, to find the Side of a Square equal to a Circle.

Multiply the Diameter by 8.862, and the Product is the Side of the Square equal.

Thus 11.3 the Diameter multiplied by 8.862, gives 10.0140 the Side of the Square equal. By Numbers thus:

The extent from 10 to 8.862, will reach the ſame way from 11.3 to 10.0148.

6. The Circumference given, to find the Side of a Square equal.

Multiply the Circumference given by 8.862 the fixed Side of the Square equal, and divide the Product by 31.416 the fixed Circumference anſwering to the Square equal, and the Quote is the Side of the Square equal required.

Thus 35.5 multiplied by 8.862, the Product is 31.4621, then this divided by 31.416, hath 10.0188 for the Side of the Square equal as before.

By the Line of Numbers.

The extent from 31.416, the fixed Circumference, to 8.862, the fixed Side of the Square equal, ſhall reach the ſame way from 35.5, the given Circumference, to 10.0188, the Side of the Square equal required.

7. The

7. The Area of a Circle given, to find the Circumference, Diameter, Square equal, or Square inſcribed.

Find the Square-Root of the fixed Area, and of the given Area; then by the Rule of Three ſay, As one Square-Root is to the other, ſo is the fixed Diameter, Circumference, Squares equal, or inſcribed, to the enquired Diameter, Circumference, Squares equal, or inſcribed.

Thus the Square-Root of 78.539, the fixed Area for a Circle whoſe Diameter is 10, is 8.862, the Square-Root of 100.288 is 10.0148.

Then as 8.862 is to 10.0148, ſo is 10 the fixed Diameter to 11.5 the enquired Diameter, and ſo is 31.416 the fixed Circumference to 35.5 the enquired Circumference, and ſo is 7.071, the fixed Square inſcribed, to 7.993, the enquired Square inſcribed, and the Square-Root of the Area is the Square equal.

Thus 31.416, multiplied by 10.0148, hath 314.4449568: Then this Product divided by 8.862, hath 35.5 for a Quotient, the Circumference required, and ſo for all the reſt.

But by the Line of Numbers, juſt as is worded in the Rule of Three.

8. Note, The Square-Root of half the Square of the Diameter, is the Side of the Square equal.

But if the Circumference be multiplied by .225078, the Product is the Side of the inſcribed Square.

Thus, .225078 multiplied by 35.5, the Circumference hath 7.9902690 for the Side of the Square inſcribed.

Alſo, Having the Superficial-content of a Circle, to find the Diameter.

Multiply 1.27324 by the Superficial-content of a Circle, the Square-Root of the Product ſhall be the Diameter, thus: 1.27324 multiplied by 282, hath 359.05368, whoſe Square-Root is 18.9487, the Gauge-Point for an Ale-Gallon for round Veſſels, and 359 is the Square of the Gauge-Point uſed in Gauging, as afterward is ſhewed.

9. To

9. To measure the Segment of a Circle.

Multiply the Semidiameter by half the length of the Arch, the Product is the Superficial-content.

Thus the Diameter being 30, the half is 15, multiplied by 15.75, the half Arch of one third part of the Circle, hath 236.4 for the Area of one third part of the Circle.

The like work serves for any Portion that goes to the Center.

10. For other Segments thus.

First find the Diameter thus:

Square half the Chord of the Segment *a, d,* and divide the Product by *d, b,* the Segments Altitude or Sine, then the Quote and Sine added is the whole Diameter of the answerable Circle.

Thus *a, d* being 9 squared, is 81; then 81 divided by 6 the Segments Altitude, *d, b* quotes 13.51, to which adding 6, makes 19,51 the Diameter *b, h.*

11. The Square-Root of the Sum of the Square of the half Segment *a, d,* and Sine *d, b,* is equal to the Chord of half the Segments Arch.

Thus, Nine times 9, *viz.* 81, and six times 6, *viz.* 36 added, is 117, whose Square-Root is 10.81.

12. To find the length of the Segments Arch very near.

Find the difference between 18 (*viz. a, c,*) the Segments Diameter, and the Sum of *a, b,* and *b, c,* the two Chords of half the Segments Arch, *viz.* 21.62, and it is 3.62, one third whereof is 1.21.

13. Then the Sum of the two Chords 21.62, and 1.21, the third part of the difference, is 22.83 the length of the Arch; then multiply 9.75, the half of 19.51 the Diameter, by 11.42 the half of the Arch of the Segment *a, b, c,* and the Product is 111.345, the whole Segment *a, b, c, g.*

From which Sum when 33.75, the Product of 9 the half Base, multiplied by 3.75 the Perpendicular of the Triangle, there remains 77.595, the Area of the Segment *a, b, c,* which was required.

14. To

14. To meaſure ſuch a Segment by the Line of Segments.

Having the Segments Altitude, and the whole Circles Area.

The extent from the Circles Diameter 19.51, counted on the Line of Numbers to 100 on Segments, ſhall reach the ſame way from the Segments Altitude 6, counted on Numbers, to .2560 on Circle-Segments.

Then the extent from 1 to 299.4, the whole Circles Area, ſhall reach from .2590 to 77.595, the Segments Area, as before on Numbers.

By the Pen thus:

Multiply the Segments Altitude by 100 the Segments Radius, by adding two Cyphers to the Segments Altitude, as here 600; then divide the Product by the Circles Diameter, and ſeek the Quotient on Numbers, and juſt againſt it on the Circle Segments, is a fourth Number, which keep.

As here 600 divided by 19.51, quotes 30.77, right againſt which on Segments is .2606.

Then the whole Area 299.4, multiplied by .2606 the Product is 78.00, near as before.

PROB. XI.

To meaſure an Elipſis.

In the meaſuring of an Elipſis, a mean Proportion Geometrical between the longeſt and ſhorteſt Diameters, is the Diameter of a Circle equal in Area to the Elipſis.

Example. Suppoſe the Diameters of an Elipſis be 20 and 15, the middle between 20 and 15 on the Numbers is 17.32; then I conclude, that a Circle whoſe Diameter is 17.32, is equal in Area to the Elipſis whoſe tranſverſe Diameter is 20, and the conjugate Diameter 15.

Then

Then having the Diameter of a Circle equal, the various cases are wrought as formerly in the Circle, into which Figure the Elipsis is thus reduced.

To draw an Elipsis to any length and bredth.

Draw two Lines for a Transverse, and a Conjugate Diameter, one Perpendicular to the other; then from the meeting Point or Center, lay off the half-length and bredth to your mind, then the half-length laid from the Point designed in the Conjugate or less Diameter, shall give a Point in the Transverse or longest Diameter at each end (being laid both ways) wherein to drive two Nails or Pins, about which Pins a Line drawn shall describe the Oval or Elipsis.

PROB. XII.

To measure a Cylender.

A Cylender is a solid Body, with sides erected perpendicular on the Circumference of a Circle, and the solidity is thus found.

Multiply the Area of the Circle, being the Base to the Cylender, by the Cylenders Altitude in Inches, and the Product is the Solid-content.

Thus the Area of the Circle being 299.4, and multiplied by the Altitude 36, hath 10778.4 Cube-Inches for solidity.

By the Line of Numbers thus:

The extent from 1 to 36, shall reach the same way from 299.4 to 10778.4, or better thus, by the Diameter and Altitude only.

The extent from 1.1284 (the Diameter of a Circle when one Inch deep is a Cube-Inch) to the Diameter, being twice repeated from the length, gives the solidity.

Thus the extent from 1.1284 to 19.51 the Diameter, laid twice the same way from 36, gives 10778 Cube-Inches, as before.

To work this last by the Pen:

Square

Square the Diameter 19.51, it is 380.03.

This Square multiplied by 36 deep, is 13682.

Then this Product divided by 1.2733, or instead of dividing by 1.2733 the Square of 1.1284, multiply by .78539 his Comp. Arith. the Square of 1.128, gives 10778, as before.

To find the Content in Gallons thus.

The extent from the Gauge-Point to the Diameter, twice repeated the same way from the length, gives the Content in like Gallons to the Gauge-Point.

PROB. XIII.

To measure a Globe, or Sphere.

1. Having the Diameter, to find the Circumference; or having the Circumference, to find the Diameter, work by the Rules last mentioned in the Circle.

2. To find the Superficial-content of a Globe round about, multiply the Diameter by the Circumference, and the Product is the Superficial-content.

Thus, The Diameter being 11-3, and the Circumference 35.5, the Product after Multiplication is 401.15.

3. To find the Solid-content of a Globe.

Multiply the Superficial-content by one sixth part of the Diameter, the Product is the Solid-content.

Thus 401.15 multiplied by 1.884, a sixth part of 11.3, and the Product is 755.76660, the Solid-content.

4. Or thus: cube the Diameter (by multiplying the Diameter by it self, and that Product by the Diameter again.) Multiply that Cube by 11, and divide this last Product by 21, the Quotient is the Solid-content of a Globe.

Thus the Cube of 11.3 is 1442.897; then this multiplied by 11, is 15871.867; then this divided by 21, quotes 755.803, near as before.

5. Or thus; Cube the Diameter, and then multiply the Cube by .5238, the ſolidity of a Globe whoſe Diameter is 1, and that Product ſhall be the Solid-content of the Globe required.

Thus, The Diameter 11.3 his Cube is 1442.897, then this multiplied by .5238, the Product is 755 .7894486, where note to cut off the right Number of Fractions.

6. By the Line of Numbers thus.

The extent from 10 a fixed Diameter to 11.3 (or any other Diameter) ſhall reach from 523.8, the ſolidity of a Globe whoſe Diameter is 10, to 755.8, the ſolidity of the Globe of 11.3, being thrice repeated.

Example. Again by Numbers.

The extent from 10 as a fixed Diameter to any other Diameter, being laid twice from 31.416, gives the Superficial-content round about.

And again, The ſame extent laid three times from 523 8, gives the Solid-content.

Thus the Diameter being 40. The extent from 10 to 40 laid twice from 31.416, gives 5026.4.

And the ſame extent laid thrice from 523.8, gives .33510: Now to inſure you of the right Number of Places, the Line of 8 Radiuſſes will inſure you at any time.

Or, as 31.416, a fixed Circumference to any other Circumference, ſo is 523.8 thrice to the ſolidity.

For, If you call the place of 523.8 Hundreds, then in turning the extent three times you paſs over the next Radius which is Thouſands, and fall on the next which is tens of Thouſands: Thus the Number is 33510, the anſwer in Cube-Inches near enough for uſe.

7. On the contrary: Having the Solid-content, to find the Diameter or the Circumference.

Divide the Space on the Line of Numbers, between the fixed Area and the given Area, into three equal parts; then that third part, laid from the fixed Diameter, gives the enquired Diameter; and from the fixed

fixed Circumference, to the enquired Circumference; and the Cube-Root of the Solid-content of a Globe, is the ſide of a Cube equal to the Globe.

Alſo note, If you take the exrent between the two ſolidities, on the triple Line of Numbers, and meaſure it on the ſingle Line, you may ſave the trouble of dividing the Space into three equal parts.

But by the Pen you muſt find the Cube-Root of both Solidities; then by the Rule of Three, ſay, As one Cube-Root is to the other, ſo is the fixed Diameter or Circumference to the enquired Diameter or Circumference.

Thus the Space from 33510 in the laſt Radius of the triple Line, to 523.8 in the firſt Radius of the ſame triple Line, being laid from 10 in the ſingle Line, gives 40 in the ſame ſingle Line, and from 31.416 to 125.6 the Circumference.

9. To find the ſolidity of the Segment of a Globe, for Copper crowns.

Firſt, Find by the Rules in a Circle Segment the Spheres Diameter, of which the Segment is part, ſo have you the Altitude of the other Segment: by taking the Altitude of the given Segment from the Spheres Diameter, remains the Altitude of the other Segment.

Alſo find the Chord of half the Segment, *viz.* *b*, *n*, or *n*, *e*.

Thus the Diameter *n*, *f*, being 40, and the Segments Altitude *g*, *n*, 10, the other Segments Altitude *f*, *g*, is 30, and the Chord of half the Segment, *viz.* *b*, *n*, 20.02.

10. Then to find the Superficial-content of the Segment, work thus:

Multiply the Square of the Chord of half the Segment, *viz.* 400.8 by 3.1416, the Product is the Superficial-content.

Or, Multiply the Circumference of the Sphere by the Segments Altitude, and the Product is the Superficial-content.

Thus, 125.66 multiplied by 10, is 1256.6, the Segments Superficial-content.

By Numbers: The extent from 1 to 20.02, being twice repeated from 3.1416, gives 1256.6, as before.

Then for the Solid-content thus:

Square the Segments Diameter, and multiply that Square by the Segments Altitude, the Product is a Parallelopipedon, whose Product keep.

Then as all the other Segment to half of the same Segment and more, one sixth of the given Segment, so is the Parallelopipedon first found to all the Squares in the Segment.

Lastly, As 14 to 11, so are all the Squares last found to the Solid-content of the Segment required.

Thus the Square of 34.645 *viz.* 1200, multiplied by 10 the Segments Altitude, is 12000, the Parallelopipedon.

Then as all *f*, *g*, 30, to (half *f*, *g*, 15, more one sixth of *g*, *n*, 10, *viz.* 1.666 in all) 16.666, so is 12000, to 6666.

Lastly, As 14 to 11, so is 6666 to 5237, the Segments Solidity required.

The two last workings is done by the Pen, as the Rule of Three, multiplying 16.666 by 12000, and dividing the Product by 30, the Quote is 6666.

Also, 6666 multiplied by 11, is 733326, which divided by 14, quotes $5237.\frac{1}{2}$.

By the Line to 8 Radiusses you may assure you of the places, but you cannot see so many Figures as by a larger Line.

11. But by the Line of Solid-Segments for a Sphere, you may find the Segments content or solidity, more easily thus:

The extent from the Globes whole Diameter counted on Numbers to 100 on the Segments, shall reach the same way from the Segments Altitude counted on Numbers to a fourth Number on the same Segments for a Sphere.

Then

Then ſecondly, By Numbers only.

The extent from 1 to the Globes Solid-content in Inches or Gallons, ſhall reach the ſame way from the fourth Number to the ſolidity of the Segment in like Inches or Gallons.

Thus the extent from 40 on Numbers to 100 on Segments, reaches from 10 on Numbers to 15.52 on the ſame Segments.

Again, The extent from 1 to 33510 on a Line of 8 Radiuſſes, reaches from .1552, a Number leſs than 1, to 5237, as by the former work.

12. Or more eaſily thus by Numbers only.

The extent from 1 to the Segments Diameter, being twice repeated from the Segments half Altitude, leſs by one eighth part, gives the near Solid-content in Inches.

Thus: The extent from 1 to 34.64, being twice repeated from 4.37, the half Altitude leſs one eighth part, gives .5240, near as before.

13. To find the Solid-content of a Globe in Ale or Beer Gallons.

The extent from 18.95, the Gauge-Point for Ale Gallons to the Diameter, or 17.15, the Gauge-Point for Wine Gallons, or 107.19 the Gauge-Point for Ale Barrels, or 113.69 the Gauge-Point for Berr Barrels to the Diameter, being twice repeated from two third parts of the Globes Diameter, gives the ſolidity in like Gallons to the Gauge-Point.

Example. Thus the extent from 18 95 to 40 the Diameter, being twice repeated from 26.666, one third part of the Globes Diameter, gives 119 Gallons.

14. To find the Segments-content in Ale or Wine Gallons more eaſily.

The extent from the Gauge-Point to the Segments Diameter, being twice repeated from the Segments half Altitude, more one eighth part, or eight and a half, gives the Content in like Gallons to the Gauge-Point.

Thus the extent from 18.95 (the Gauge-Point for an Ale Gallon) to 34.64 the Diameter, being laid twice from 5.63 the Segments half Altitude, more one eighth, gives 18 Gallons 80 Parts.

Or again, By Mr. *Anderſons* example; the Segments Diameter 180, and the Altitude 54, and the Content in Cube-Inches 769824, being 2730 Beer Gallons.

The extent from 18.95 to 180 the Segments Diameter in Inches, being twice repeated from 30.17, the Segments half Altitude, more one eighth part and a half, to 2730 Gallons, near as before.

But in Spheriods Segments, count a Geometrical mean Proportion between the longeſt and ſhorteſt Diameters, and then work as before in the Segments of a Sphere, but the Spheriod-Segments on the Rule is more quick and ready, and as exact.

PROB. XIV.

To meaſure a Cone and its parts.

A Cone is a round Pyramid, and meaſured in the ſame manner as the ſquare Pyramid, by multiplying the Area of the Baſe by one third part of the Altitude, and the Product is the Solid-content in like meaſure.

Example. Let the Diameter of the Baſe of a Cone be 60 Inches, and the Perpendicular Altitude 98, found by meaſuring, or by the Rule in the Pyramid, the Area of a Circle whoſe Diameter is 60, is 2828 Inches, found by Prob. 10.

Then this multiplied by 32.66, one third of 98 the Altitude, hath 92362 the Solid-content of the Cone in Cube-Inches.

By Numbers thus: The extent from 1 to 2828, the Area of the Baſe, being laid the ſame way from 32.66, one third of the Altitude, gives 92362 Cube-Inches for the Solid-content.

Or

Or rather thus: The extent from 1.128 (the Diameter of a Circle, when one Inch deep is a Cube-Inch) to the Diameter, laid twice the same way, from one third of the Altitude, gives the Solid-content.

Thus: the extent from 1.128 to 60 the Diameter, being twice repeated from 32.66, one third part of the Altitude 98, gives 92362 the Solid-content of the Cone in Cube-Inches.

By the Pen this is wrought thus:

1. Multiply the Diameter 60 by 32.66, and the Product is 1959.6000: then this Product divided by 1.128, quotes 1736, a fourth.

2. Multiply this fourth 1736 by 60, and the Product is 104160, and this Product divided by 1.128, quotes 92362.

Or else thus by the Pen:

Square the Diameter 60. which is 3600, multiply this Square-Number by 32.66, one third of the Altitude, the Product is 117576,00.

Then this Product divided by the Square of 1.1284, *viz.* 1.273240, the Quotient is 92362 as before.

Or, Multiply the Square of the Diameter by one third of the Altitude, and multiply that Product by 7854 the Arithmetical Complement of the Square of 1.128, found by laying the extent from 1 to 1.128, twice back from 1 to .7854.

But the Rule and Compass is ten times quicker. Therefore note it here well, to avoid repetition afterwards.

For the content in Beer or Wine Gallons, or Beer or Ale Barrels, thus:

The extent from the Gauge-Point to the Cones Diameter at Base, being twice repeated from one third of the Altitude, gives the Solid-content in Gallons or Barrels, as the Gauge-Point was.

Thus in this Example: The extent from 18.95 to 60, laid twice from 32.60, reaches to 327 Ale Gallons and a half, as the Gauge-Point was.

Or

Or by the Pen. Square the Diameter, and multiply that Square by $\frac{1}{3}$ of the Altitude, then divide that Product by the Square of 18.95, the Gauge-Point, *viz.* 359: the Quote is the Gallons or Barrels as the Gauge-Point was.

To measure the Frustum, or lower part of a Cone, as round Tuns are.

First by the difference of Diameters, to find the length of the whole Cone, whereof this lower part is a Frustum.

As the difference of Diameters to the Frustums Altitude, so is the greater Diameter to the whole Cones Altitude.

Thus: Let the greater Diameter be 60 Inches, and and the lesser Diameter 36.5, and the Frustums Altitude 38.4.

Then the extent from 23.5 the difference of Diameters, to 38.4 the Frustums Altitude, shall reach from 60 the great Diameter, to 98 the whole Cones height.

Then 38.4 the Frustums Altitude, taken from 98 the whole Cones Altitude, remains 59.6 the height of the lesser Cone; then the solidity of the lesser Cone, taken from the whole Cone, remains the solidity of the Frustum Cone, or Tun. This is a true and natural way of measuring, otherwise thus as followeth:

1. Multiply the greater Diameter by the lesser Diameter, the Product is called the Rectangle of the Diameters.

2. Also add the two Diameters together, and then square the Sum of them, and note the Product.

3. Substract the Rectangle of the Diameters, from the Square of the Sum of the Diameters, and note the Residue.

4. Multiply this Remainder, by the depth of the Tun in Inches, noting the Product.

5. Then

5. Then this last Product divided by 1077, gives in the Quotient the number of Ale Gallons required.

Or divided by 882, gives the Solid-content in Wine Gallons.

Or by 3.8194, gives Cubical-Inches.

Example. 1. The greater Diameter 72, the lesser 58 multiplied, makes 4176 for the Rectangle.

2. The Sum of 58 and 72, *viz.* 130 squared, is 16900.

3. The Rectangle 4176 taken from 16900, remains 12724.

4. Then this Residue multiplied by the depth 43, the Product is 547132.

5. This Product divided by 1077, the Quote is the Content in Ale Gallons, *viz.* 508 $\frac{16}{1077}$ the true Content.

Or divided by 882, gives in the Quote the Number of Wine Gallons 512.620 Wine Gallons.

Or by 3.8194 gives the Content in Cube-Inches, *viz.* 143256 Cube-Inches.

Or more easily thus, as a Cylender is measured: as thus for Example.

Add the two Diameters together to find the Sum, and substract one from the other to find the difference between them, then count the half-Sum as a mean Diameter, note also the half-difference, and one third of the depth, in this method and manner.

Then the extent from 1.128 to the half Sum of the Diameters 65, being twice repeated from the depth 43, gives 142706 Inches.

Again, the extent from 1.128 to 7, the half difference, being twice repeated from 14.33, one third of the length, gives 550: then these two added, makes 143256 Cube-Inches the Area.

Or, the extent from the Gauge-

Diam.	72
	58
Sum—	130
$\frac{1}{2}$ sum	65
Differ.—	14
$\frac{1}{2}$ differ.	7
Depth	43
$\frac{1}{3}$ depth	14.333

Point

Point 18.95, to 65 the Arithmetical mean Diameter, laid twice from 43, gives 506.3 Gallons +, then the extent from 18.95, to 7 the half difference of Diameters, twice repeated from 14.333, one third of 43, the length gives 2 Gallons, in all 508 the Content. This is a good way, and eaſily remembred.

CHAP. XIII.

A brief rehearſal of the former Problems, and Applications obſervable depending thereon.

1. IF a Cube be made, or conceived to be made, whoſe ſides are 12 Inches every way, then the Solid-content is 1728 Cube-Inches.

2. If a Priſm have the ſame Baſe and Altitude, his Solid-content is 864; for 144 multiplied by 6 half 12, hath 864 for a Product.

3. If a Pyramid have the ſame Square, Baſe and Altitude 12, his ſolidity is 576; for 144 the Area of the Baſe, multiplied by 4 a third of 12, gives 576 for the Solid-content of the Square-Pyramid.

4. If a Cylender be made of 12 Inches Diameter, and 12 Inches high, the Solid-content will be 1357$\frac{1}{7}$, for the Area of a Circle, 12 Inches Diameter is 113.12, then this multiplied by 12, produceth 1357.54.

5. If a Triangular Pyramid be made to have each ſide of his Baſe 12, and his Perpendicular 12 alſo, his Solid-content is 294.6: for the Area of an equilateral Triangle whoſe ſide is 12, is 62.4, and that multiplied by 4 a third of 12, hath 294.6 for the Solid-content of the Triangular Pyramid.

6. If a Cone be made whoſe Diameter is 12, and his Perpendicular Altitude 12, his Solid-content is 452.8 for the Area of the Baſe being the ſame with the Circle

or

or Cylender of 12 Inches Diameter, being multiplied by 4, one third of the Altitude 12, hath 452.8 for his Solid-content in Inches.

7. If a Sphere or Globe be made 12 Inches in the Axis or Diameter, the Solid-content by the former Rules will be found to be 905-14.

8. Note, that the Superficial-content round about a Cylender, is equal to the Superficial-content of a Globe, whose Axis is equal to the Cylenders Diameter and Altitude, (abating the two Circles at the top and bottom of the Cylender.)

Or the same again briefly thus:

	Cube Inches.
I. Cubes solidity whose side is 12	1728
1. Cylenders solidity is	1357—72
II. Spheres solidity	905—14
2. Prisma's solidity is	864
III. Square Pyramids	576
Triangular Pyramids	249—5
3. Cones solidity is	452—8

By the foregoing Proportions it is observable that a Cube is double the Prisma, and triple to the Square-Pyramid of equal Base and Altitude, or as 3.2 and 1, for three times 576 is 1728, and two times 864 is 1728 also.

Further note, A Cone is one third of a Cylender, and a Sphere is two third parts of a Cylender, or two Cones is equal to a Sphere, and three Cones equal to a Cylender of the same Base and Altitude; as 1, 2, & 3.

Also: A Cylender is $\frac{11}{14}$ of a Cube, and a Sphere is $\frac{11}{21}$ of a Cube, or two thirds of a Cylender.

Therefore as 2 to 1, so is 1728 the Cube to 864 the Prism.

Or as 1 to 2, so is 864 the Prism to 1728 the Cube.

Again, as 1 to 3, so is 576 the Square-Piramis to 1728 the Cube.

Or

Or as 3 to 1, so is 1728 the Cube to 576, the solidity of the Square-Pyramid.

Also: as 11 to 14, so is 1357.72 the Cylender, to 1728 the Cube.

Or as 14 to 11, so is 1728 the Cube, to 1357 the Cylender.

Again, as 3 to 2, so is 864 the Prisma, to 576 the Square-Pyramid.

Or as 2 to 3, so is 576 the Square-Pyramis, to 864 the Prisma.

Lastly, As 21 to 11, so is 1728 the solidity of a Cube, to 905.14 the solidity of a Globe, and the contrary.

Therefore having the solidity of the more easie Solids, by these easie Proportions you may find the more difficult, and prevent mistakes thereby.

Thus you have an acconnt of the usual Solid-Bodies, into which other more Irregular-Bodies are reduced, in order to the mensuration of them.

In this second Part is the more particular measuring of close and open Vessels, used by Brewers and Coopers, in the plainest and shortest ways yet in use.

1. As by the Digonal-Lines.

2. By *Oughtreds* Gauge-Lines, being Lines with equal parts annexed, to give the whole Area, or one third of the Area of a Circle at any Diameter, and one Inch deep.

3. By the Line of Numbers, with the equated Diameters and Gauge-Points.

4. By Arithmetick, and several Abreviations. By Multiplicators and by Divisors.

CHAP.

CHA

To find a mean Diam
Squelling Cask to a C

Multiply the difference
vide the Product by
lesser Diameter, and the

Exam le. Let the Bun
Head 24, the difference is
viz. 4 Inches 2 10*th* to a
tenth, the mean Diameter.

Or, Multiply the differe
the Product under the gr
forwarder to the right hand
set, substract it from the
Residue is the mean Diame

Greater Diameter-

Three times 6 the Differ
ward one place, and taken

Another way fit

Multiply the lesser Diame
of Diameters, and to the P
the difference.

Then deduct one third pa
Square of the Bung Diame
Square of the equated Diame
of this Square is the equated

F

HAP. XIV.

Diameter that ſhall reduce to a Cylender near the truth.

erence of Diameters by .7, and
uct by 10, the Quote add to
nd the Sum is the mean Diamete
he Bung Diameter be 30, and
ence is 6; then ſix times 7 is
th to add to 24, the Sum is 2
ameter.
difference of Diameters by 3 : :
the greater Diameter, one pl
nt hand than is uſual, and being
m the greater Diameter, and
Diameter thus :

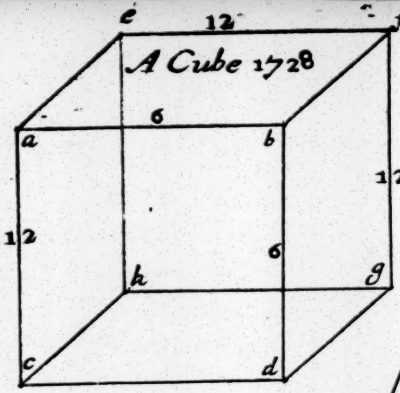
r---------- { 30—
{ 1—

e Difference is 18 ſet for- { 28—
taken from 30, reſt

way fit for a Spheroid.

r Diameter, by twice the differe
o the Product add the Square

third part of this Sum, from
Diameter, the Reſidue is
d Diameter, and the Square-R
equated Diameter.


e
12
f
A Cube 1728
6
a
b
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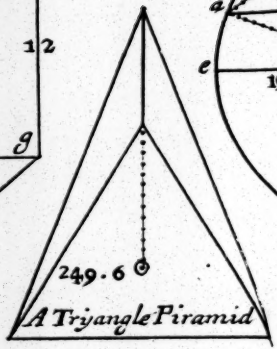
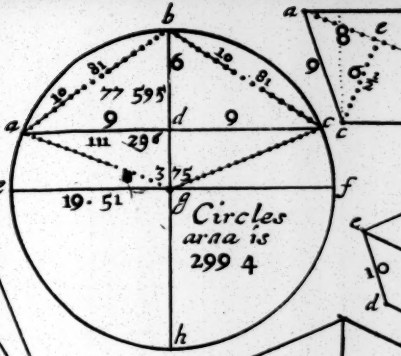
h
g
c
d


249.6
A Triangle Piramid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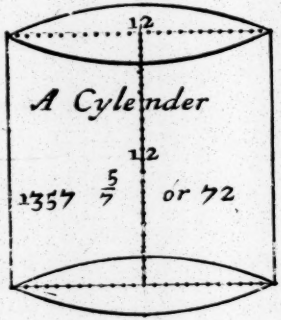
12
A Cylender
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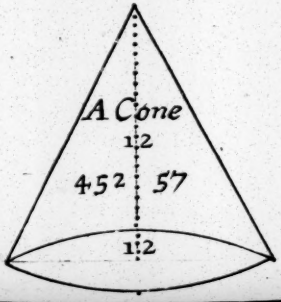
1357 5/7 or 72


A Cone
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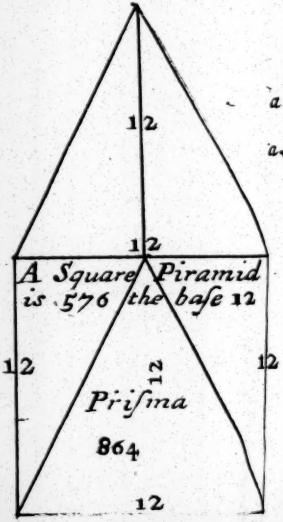
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A Square Piramid
is 576 the base 12
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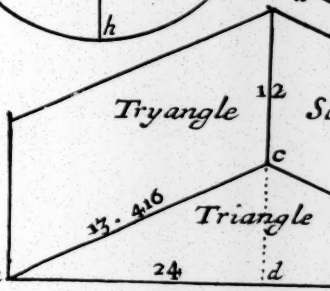
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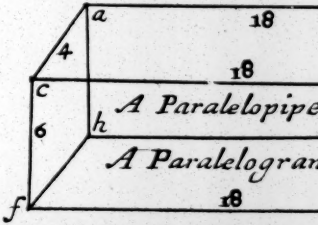
a
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c
A Paralelopipe
h
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A Paralelogram
f
18


32
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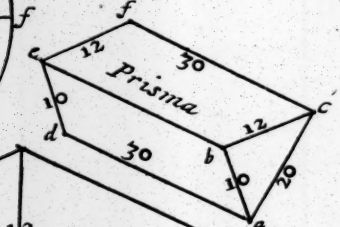
Prisma
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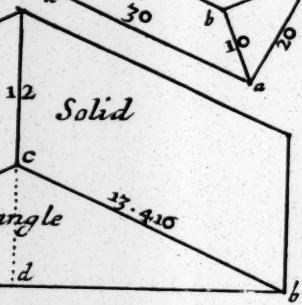
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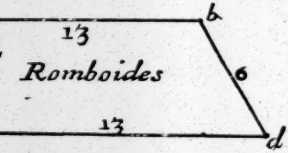
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Romboides
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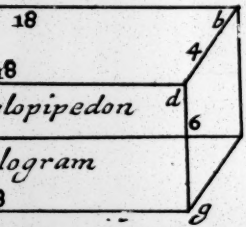
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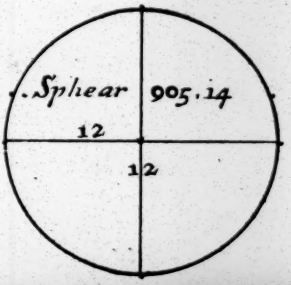
Sphear 905.14
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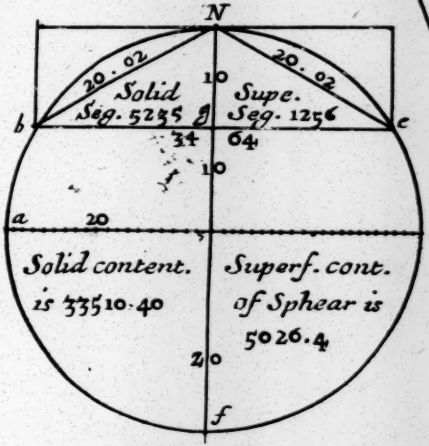
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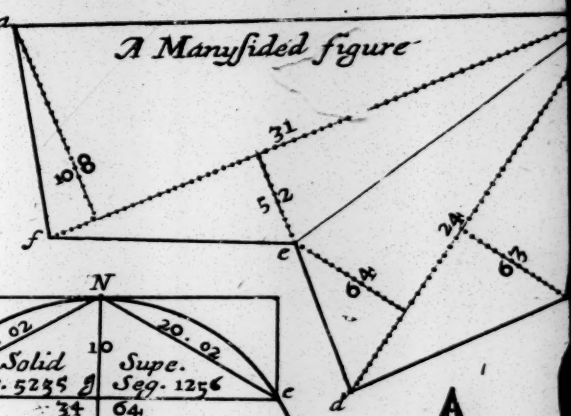
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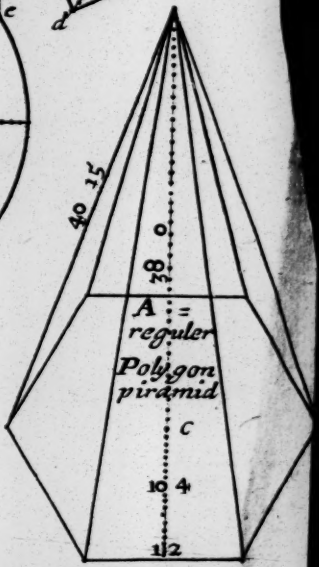
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Or, Seek the leſſer Diameter on true Inches, on the Gauge-Line is one third of the Area, which ſet down once: ſeek alſo the greater Diameter, and in the Gauge-Line is that Area, to ſet down twice: then add them in one Sum, and ſeek it in the Gauge-Line, and right againſt it in the ſmall Inches is the true mean Diameter for a Spheroid.

Of Gauging of cloſe Casks.

CHAP. XV.

To find the Content of any cloſe Veſſel ſeveral ways.

CLoſe Casks (by ſeveral Gaugers) are ranged into five ſorts, *viz.* Cylendrical, Spheroidal, Parabolical, Conoidal and Conical; which five ſorts are beſt diſtinguiſhed one from the other by this Figure.

Let *a, b, c, d, e,* repreſent a Graves Hogſhead, *g, h,* the length 32.06, *b, l,* the Bung Diameter 25, *c, d,* or *a, e,* the Head Diameters within the Cask 22 Inches, the Diagonal *b, d,* and *b, e,* 28.40, but the Diagonal-Line *b, f,* as Cylendrical 29.65.

1. The Veſſel as Cylendrical, with Head and Bung the ſame in Diameter, as the pricked Line and Name Cylendrical ſheweth.

2. The Veſſel as Spheroidal, the Arch-Line *a, b, c,* plainly ſheweth, with the Name to it.

3. The

3. The Veſſel as Conical, repreſented by the pricked Line *a*, *b*, *c*, ſtraight from *a*, to *b*, and ſo ſtraight again from *b*, to *c*, being a Segment of two Cones, joined upon one common baſe *b*, *l*.

4. The Veſſel as Parabolical, lies between the Conical and Spheroid, very hard to be expreſſed, and more hard to be diſcovered in the gauging thereof, as the Line *o*, *p*, may ſhew below the other.

5. As Conoidal, as the Line *m*, *n*, ſheweth.

1. To gauge this Cask as Cylendrical.

By the Diagonal-Line, put the braſed end of the Gauging-Rod into the Bung-Hole of the Cask, with the Diagonal-Lines upwards, and thruſt the braſed end to the meeting of the Head and Staves, as upright as you can, diagonally as the Line *b*, *d*, in the Figure.

Then with Chalk make a mark on the middle of the Bung-Hole of the Veſſel, and alſo on the Diagonal-Lines on the Rule, right againſt, or over one another, when the braſed end is thruſt home to the Head and Staves.

Then turn the Gauging-Rod to the other end of the Veſſel, and thruſt the braſed end home to the Head as before.

And ſee if the mark made on the Gauging-Rod come even with the mark made on the Bung-Hole, when the Rod was thruſt to the other end:

Which if it be, then the mark made on the Diagonal-Lines, ſhall on the ſame Lines ſhew the whole Content of the cloſe Cask in Wine or Beer Gallons.

But if the mark firſt made on the Bung-Hole, be not right againſt that made on the Rule, when put the other way; then right againſt the mark made on the Bung-Hole, make another on the Gauging-Rod, on Diagonal-Lines; then the diviſion on the Diagonal-Lines, between the two Chalks, ſhall ſhew the Veſſels whole Content in Wine or Ale Gallons.

Example. In this Veſſel, the Diagonal-Line *b*, *f*, for the Cylendrical Veſſel, will be found 29 Inches 65

Parts, right against which on the Diagonal-Lines is 57 ½ Beer Gallons, or 70 Wine Gallons: and so much is the whole Content of such a Cask, as Cylendrical by the Diagonal-Lines.

But if the Vessel be Spheroidal, as the Diagonal-Line *a*, *d*, sheweth, then the length of the Diagonal-Line is but 28 Inches 4 Tenths, which gives on the Diagonal-Lines but 51 Beer Gallons near, or 62 Wine Gallons, the Content of the Vessel as a Spheroid by the Diagonal-Lines; which Diagonal-Lines affords the whole Content of close Casks, made of a usual form to 48 Inches in the Diagonal-Line, being 246 Beer, or 300 Wine Gallons.

But if the Vessel be open, as half a Barrel, or a Tun, or a Copper, and the measure from the middle on one Side, to the Head and Staves be 38 Inches, the Diagonal-Line shews 122 Beer Gallons, the half of 122, *viz.* 61, is the Content of the open half Tub.

But if you would try a large Vessel, as a Tun, or Copper, and the Diagonal-Line taken by a long Rule, prove 70 Inches, then by the small divisions at the beginning of the Diagonal-Line, you may attain your desire thus:

Every Inch at the beginning-end call 10 Inches, then 10 Inches becomes a 100 Inches.

And every tenth of Gallon call 100 Gallons on the Diagonal-Line, and every whole Gallon with a Figure call 1000 Gallons.

Example. At 44 Inches, and 8 Tenths, on the Diagonal-Beer-Line is 200 Gallons: so also at 4 Inches 48 Parts, called now 44 Inches and 8 Tenths, is just 2 Tenths of 1 Gallon, now called 200 Gallons.

Also, If the Diagonal-Line prove 76 Inches and 7 Tenths, a close Cask of so great a Diagonal, holds 1000 Beer Gallons, but an open Cask but half so much, *viz.* 500 Beer Gallons.

Thus much for the use of the Diagonal-Lines to measure Vessels.

But

But for reducing of Wine Gallons to Beer Gallons, or Beer Gallons to Wine Gallons, a look of your Eye from one Line to the other is enough.

Thus: 30 Wine Gallons, is 24 $\frac{1}{2}$ Beer Gallons, and 60 Beer Gallons, is 73 $\frac{1}{3}$ of Wine Gallons, and the like.

Now followeth the neareſt, and readieſt ways yet in uſe, and illuſtrated by five Examples, *viz.* Of a Cylender, a Spheroid, a Parabola, a Conoidal, and a Conical Veſſel, the work being near the ſame in all four, excepting the conſideration for the alteration of the form of the Veſſel, as to the ſtraitneſs and ſwelling of the Staves or Sides.

An Adverti ſement as to taking the dimentions of cloſe Casks.

To do this work compleatly, and readily:

1. There ought to be three Rules uſed, one to uſe in the Bung-Hole only, and from thence removed from Veſſel to Veſſel, to avoid ſpilling or ſoiling your Hands or Cloaths, with the Oil or Wine meaſured.

2. Another ſhorter, to meaſure the Heads only, with a ſliding Furral.

3. A long ſliding Rule, with two Hooks or Returns at each end, to ſtay again the two Heads, on which you may have allowed a competent and experimental allowance for the thickneſs of the two Heads, by a piece to ſlide on the Hooks.

But theſe three are abſolutely neceſſary for a Gauger at the *Cuſtom-Houſe-Key*, or ſuch like place, where much work is to be done in a little time.

But to perform this by one Rule, I may adviſe thus:

Firſt, For the Diameter at Head.

Apply the brazed end of the Rule as cloſe to the Chines as you may, then the greateſt Diameter take a-croſs in ſeveral places, and equated if you find a difference, is the true Diameter at the Head of the Veſſel,

which ought to be tried at both ends, leſt there ſhould be a difference in the Heads: if there be, then the Arithmetical mean or half-Sum of both the Heads, is the Diameter at the Head.

To find the Bung Diameter.

The Hole being open, then put the brazed end downright to the oppoſite Staff, and the inſide of the Bung-Hole gives the Bung Diameter.

But if the Veſſel be preſſed ovalliſh, as ſometime lying upon one another they may, you muſt allow for it, or elſe find the Diameter on the outſide with a large pair of Callipers, or by two Perpendicular-Lines juſt touching the Sides, or two Horizontal-Lines, one touching the bottom, the other the top of the Veſſel.

And if the Bung-Hole may not be opened, a ſtrait Rule or Line extended along the length of the Cask, juſt to touch the Bung, the diſtance between the Line and Chines at both ends being equal, twice that difference added to the Diameter at Head, is the true Bung Diameter.

To find the length of a Cask.

Meaſure from the Head to the extremity of the Chine over-hanging, and lay the ſame extent on the Hoops.

Then there ſtick up a Knife, allowing what you think fit for the Head.

Do the like at the other end, and ſtick up a Knife there alſo upright, with allowance for that Head alſo.

Then the exact meaſure between the two Knives, is the length of the inſide of the Veſſel.

If you mind to be exact, turn the Veſſel, and meaſure the length on the oppoſite ſide alſo.

If they differ, uſe the Arithmetical mean.

1. To gauge any Cylendrical-Veſſel by the Pen, without Tables or Lines.

Square the Diameter, then multiply that Square by the length of the Veſſel, then multiply this laſt Product by the Multiplicators in the Table, and you ſhall have the

the content in Cubical-Inches, Wine or Ale Gallons, Beer or Ale Barrels, according to the Multiplicator used.

Example. In this Graves Hogshead, the Square of 25 the Diameter, is 625; this Square multiplied by the length 32.06, produceth 20037.50; then this Product multiplied by 0027851, gives 55 Gallons 80644250 Parts of a Gallon.

Or being divided by 359.0533, quotes the same Content in Gallons of Ale, *viz.* 55.8064, likewise being multiplied by .78539, or divided by 1.127324, doth add-eth or quotes the Content in cubical Inches.

Part to follow the method used in the four Examples following,

Set the Square of the Diameter three times down; for the Bung-Diameter twice, and the Head once. { 625 / 625 / 625 }

Then add the three Squares into a Sum, which is ——— { 1875

Then this multiplied by the length ——— 32.06

produceth .6011250.

Lastly, This Product multiplied by 000928366, produceth 55 Gallons, 8064, *&c.* as before.

Or being divided by 1077.16, quotes 55 Gallons 806.

2. By the Gauge-Line to measure this Cylendrical Vessel in Ale Gallons.

Seek for the Diameter of the Vessel in the small Inches, and just against it on the Gauge-Line is the quantity of Beer Gallons contained in one Inch deep; then this Sum multiplied by the depth or length in Inches gives the true Content in Beer Gallons.

Exam-

Example.

	Gal. Par.
Right againſt 25 in ſmall Inches on the Gauge-Line,, is	1.745
Which multiplied by the length	32.06
Gives the Content in Ale Gallons, *viz.*	55.9447.

3. By the Line of Numbers thus:

The extent from 18.95, the Gauge-Point for an Ale or Beer Gallon, to 25 the Diameter, being the ſame repeated from 32.06 the length, reaches to 55.9 juſt one Content as before.

4. If you ſeek for 25 on the true Inches, right againſt 25 is 0.589, on the Gauge-Line for Beer, being the third of the Area, and three times this, *viz.* twice for the Bung, and once for Head, is 1.740, near as before by the whole Areas, which being multiplied by 32.06, produceth 55.7844, near as before for the Content of ſuch a cloſe Cask as Cylendrical. And this way ſerves for all open Veſſels, having gained the mean or equated Diameter, between the top and bottom, or greater and leſſer Diameters.

For the more ready gaining of the equated Diameter, in all ſorts of cloſe Casks, being taken under any of theſe four notions, *viz.* Spheroidal, or the middle Segment of a Spheroid.

Or as a Parabolical Spindle, which for brevity ſake, I call a Parabola.

Or as a Conoid, or middle Segment of two Conoids, abutting on one common Baſe, whoſe middle at the Bung is ſomewhat round and ſwelling, but towards the two Heads the Staves run ſtraight.

Or as two Cones abutting on one common Baſe, whoſe Staves run directly ſtraight from the Bung towards each of the Heads, as is partly ſeen in the Figure of the Graves Hogſhead, by the Lines, with theſe names written on them.

For

For this use of finding the equated Diameter is four Scales (of near equal parts) drawn on the printed Paper, and may be set on Gauging-Rods, next the true Inches, whose use is thus.

When you are resolved under what notion to gauge your Vessel, whether as Spheroid, or Parabola, or as a Conoid, or as a Cone; and having taken the Dimensions of the Cask at Head and Bung in Inches, then seek the difference of Diameters on that Line so named, as you do account your Cask to be, and just against it in the Inches, are the Inches and tenth Parts to add to the Diameter at Head, to make the true equated Diameter required.

Four Examples of this Graves Hogshead, under the four notions following, that of a Cylender being already done withall.

1. And first as a Spheroid by the Pen.

Square the Bung-Diameter, and set the Square down twice.

Square the Head-Diameter, and set that down once.

Then add the three Squares into one Sum, then multiply that Sum by the length of the Vessel, and then multiply this Product by .00092837, the Product is the Content in Ale Gallons: Or if you had divided by 1077.16, then the Quote would have been the Content in Ale Gallons. What the Multiplicator and Divisor is, in the Table following you may see.

Thus the Sqaare of 25, the Bung-Diameter is	625 625
The Square of 22, the Head, is	484
The Sum is	1734
The length of the Vessel is	32.06

The

The Product after Multiplication, is 55592.04; and the second Product when multiplied by 00092837, is 51.60994504; or the Quote after dividing by 1077.16, is 51.6099, as before by Multiplication.

2. By the Lines on the Gauging-Rod.

Seek 25 the Bung-Diameter, on the Line of Inches on the Rule, and just against it on the Gauge-Line for Ale Gallons, is 0-580:

Set this down twice	0.580 0.850
Just against 22 Inches, the Head-Diameter on the Gauge-Line is	0.4496
The Sum is	1.6096

Which multiplied by 32.06 the length, the Product is 51.603776, the Content in Ale Gallons.

3. By the equated Diameter and Line of Numbers.

Look for 3 Inches, the difference of Diameters, on the Line called Spheroid, and just against it on the Inches is 2 Inches and near $\frac{1}{2}$, a tenth, to add to 22 the Diameter at Head, to make a mean Diameter, *viz.* 24 Inches 05. Then say,

The extent from 18.95, the Gauge-Point for Ale Gallons, to 24.05 the mean or equated Diameter, being laid twice the same way from 32.05, the length shall reach to 51.61, the Content in Ale Gallons, as before.

4. Or just against 24.05 on small Inches on the Gauge-Line, is 1.610, which multiplied by the length, gives 51 Gallons 60 Parts.

Note, This Rule serves for all swelling Casks, whether Tuns, Coppers, or other Vessels, whose Sides are arched outward, and hollow inward, and therefore called Spheroids, as the Figure of this Graves Hogshead is.

2. A

2. A second Example of the same Vessel, counted as a Frustum, or middle Section of a Parabolick Spindle. By the Pen:

Square the Diameters of Head and Bung, set the Square of the Bung-Diameter down twice, and the Square of the Head once, and add them together in a Sum.

Square also the difference of Diameters, and multiply it by 04, (or 4 Tenths,) and substract this Product from the Sum abovesaid; then multiply the Remainder by the length of the Vessel, then this last Product divided by 1077.16, or multiplied by 00092837, quotes or produceth the Content in Ale Gallons.

The Sum of the three Squares, is as in the first Example before. The Square of 3 the difference of Diameters, is 9, which multiplied by 04, produceth 3.6, which taken from 1734, remains 1730.4. Then this Remainder multiplied by 32.06, produceth 55476.624, which divided by 1077.16, quotes the Content in Ale Gallons, or multiplied by 00092837, produceth 50.50283342 Gallons.

	1734
	3.6
	1730.4
	50.503

2. By the Lines on the Rule as before.

Seek for 25 on Inches, and right against it on the Gauge-Line for Ale, is	0.580
Which set down twice. Against 22 Inches on the same Line, is	0.580
	0.4496
The three Sums added, are	1.6096
Then seek for 3 Inches, the difference of Diameters on the Inches, and right against it on the Gauge-Line for Ale is .00084, and 4 times this, is 00336, which taken from 1.6096, remains 1.5760.	.00336
	1.5760

Lastly, This Remainder multiplied by 32.06, the length gives 50 Gallons, 42656 Parts.

3. By the equated Diameter and Numbers.

Just against 3 Inches, the difference of Diameters sought on the Line Parabola, is 1.80 on the Inches, which

which add to 22, makes 23.80 an equated Diameter, (or as 10 to 6, ſo is 3 to 1.80, as before.)

Then the extent from 18.95, the Gauge-Point, to 23.80 the equated Diameter, laid twice from 32.06, gives 50.51.

4. Or, Having the equated Diameter 23.80, ſeek it in the ſmall Inches, and right againſt it in the Gauge-Line, is 1.575, which multiplied by 32.06, gives 50 Gallons and 4995 Parts for the Content in Ale Gallons.

3. A third Example as a Conoid, or the middle Fruſtum of two Conoids, and firſt by the Pen, from *M. D.*

Square the Bung and Head-Diameters, as before, and ſet them once down, and add them together in one Sum, and multiply that Sum by once and a half the length, the Product divide by 1077.16, the Quote is the Ale Gallons.

The Square of 25, the Bung is	625
The Square of 22, the Head-Diameter	484
The Sum added, is	1109

32.06 the length once and a half, is 48.09:
The Product after Multiplication, is 53331.81.
Then this Product multiplied by 00092837, produceth 49 Gallons, 51165, *&c.* or divided by 1077, quotes 49 Gallons 51165, the true Content.

2. By the Lines on the Rule thus.

Seek the Bung-Diameter 25 Inches on the Line of Inches, and right againſt it on the Gauge-Line for Ale meaſure, is 580 / 580

Againſt 22 the Head-Diameter, is	4496
The Sum when added, is	1.6096
~~8 times 00084, found againſt 3 Inches, the difference of Diameters on Gauge-Lines, is~~	00672
~~Which being ſubſtracted, the Remainder is~~	1.5424

Then

Then right against 3 the difference of Diameters, is 00084, and 8 times this, is 00672, which taken from 1.6096, remains 1.5424, which multiplied by 32.06 the length, produceth 49 Gallons 449344.

3. By the equated Diameter and Numbers.

Seek for 3 Inches, the difference of Diameters, on the Line called Conoid, and just against it on Inches is 1.55 to add to 22 the Diameter at Head to make a mean Diameter.

Or without those Scales. Say, as 10 to 5.18, so is the difference 3 to 1.55 to add to 22.

Then having the equated Diameter, the extent from the Gauge-Point to the mean Diameter, being twice repeated from 3206, the length gives 49 Gallons .50.

Or count the equated Diameter on the small Inches, and right against it on the Gauge-Line, is 1.54, which multiplied by 32.06, produceth 49.38 Ale Gallons.

4. A fourth Example, to gauge the same Vessel taken as Conical, *viz.* the middle Frustum of two Cones, abutting one against the other on one common Base.

Add the Diameters at Head and Bung into one Sum, and square that Sum.

Also, Multiply the two Diameters, one by the other, to find the Rectangle or Product of them.

Then substract the Rectangle, or Product of the two Diameters, from the Square of the Sum of the Diameters, and multiply the Remainder by the length, this last Product multiply by 00092837, or divide it by 1077.16, and the Product or Quore gives the Content in Ale Gallons.

Thus the Sum of 25 and 22, is 47, and the Square of 47, is 2209. The Product or Rectangle of 25, multiplied by 22, is 550; this Rectangle taken from 2209, rests 1659; then this Remainder multiplied by 32.06 the length, the Product is 53187.54; then this Product multiplied by 00092837, the Product is 49 Ale Gallons 3777155 Parts.

2. To do this by the Gauge-Lines on the Rule.

Seek the Bung and Head-Diameters on Inches, and on the Gauge-Line for Ale, is 0.580.

Againſt 25 Inches, which ſet down twice	0.580
	0.580
Againſt 22 Inches the Head-Diameter, is	0.4496
The Sum is when added together	1.6096
Then againſt 3 Inches, the difference of	0 0756
Diameters, is 00084.9 times this is 00756,	
which taken from 1.6096, remaines 1.5340,	1.5340

which multiplied by the length, produceth 49 Gallons 18 Parts, ſomewhat leſs than before, (therefore multiply 00084 by 8.5.)

3. By the Gauge-Line and ſmall Inches, thus:

Seek 25 the Bung-Diameter on the ſmall Inches, and right againſt it on the Gauge-Line for Ale, is 1.741, being read as the whole Area of a Circle. Right againſt 22 on ſmall Inches, on the Gauge-Line for Ale, is 1.348, the Sum of them when added, is 3.089.

Then ſeek the difference of Diameters on the true Inches, and on the Gauge-Line, is 00084, being read as thirds of Areas; which Sum taken from 3.089, reſts 3.0806, which being multiplied by 16.01, the half length, produceth 49 Gallons, 3204 Parts.

4. By the equated Diameter and Gauge-Point.

Seek for 3 the difference of Diameters on the Line called Cone, and juſt againſt it on Inches, is 1.5, to add to 22 the Head-Diameter, to make an equated Diameter.

Then the extent from 18.95, to 23.5, laid twice from 3205, gives 49 Gallons 40 Parts.

Or without thoſe Lines; ſay, As 10 to 5.15, ſo is 3 the difference of Diameters to 1.54, to add to the Head-Diameter, to find a mean or equated Diameter.

By theſe four Examples you may ſee, that Mr. *Windgates* way of adding of 7 Tenths, is too much in Conoidal and Conical Veſſels, and in very much ſwelling

Veſſels

Veſſels too little, ſo that between 7 Tenths and $\frac{1}{2}$, and 5 Tenths, will all Veſſels happen to be that are hooped.

And by diligent Practice and Experience, the Practical-Gauger may know what part to make uſe of, when he ſees his Veſſel before him that is to be gauged.

Theſe four Examples are reſolved in Ale Gallons: but in the following Table is the Multicators and Diviſors for Wine Gallons, or Cubical-Inches, for Wine or Ale Pints, for Ale or Beer Barrels, and for Corn and Sea-Coal Meaſures,

Theſe are the beſt and brieſeſt ways yet in uſe, and in this method here uſed, the Gauger hath three or four ways to uſe, which he pleaſeth.

Next followeth the Table of Cube-Inches, Gauge-Points, Diviſors and Multiplicators, and the way to make them on the ~~back-ſide~~ next pag of the Table.

Where note, The Numbers in this Table are contracted to avoid tediouſneſs in Multiplication or Diviſion, and by that means may not arrive at ſuch exactneſs and agreement one with another, as they would if more Figures were uſed.

Names of the Veſſels.	Multiplicators being Compl. Arith. of the laſt Divif. for round Tuns.	Diviſors bein trip. the other Diviſors uſed in Conical round Tuns.	Multiplicators uſed after ſquaring the Diam. being Co. Arit of the Diviſors f. whole Cones	Diviſors uſed after ſquaring the Diameters being Square of Gauge-Poi In 1000.	Cube-Inches in the Veſſels, as Diviſors. In 1000.	Gauge-Poi. for round Veſſels, or Diam. Circl. In 1000.	Gauge-Poi. for Square Veſſels or Squar. Root. In 1000.
For Cub. Inches.	0 .26188	3.81972	.78539	1.27324	1.0000	1.12838	1.0000
Wine Pint.	0 .0090525	110.30173	.0271999	36.767245	28.8750	6.0636	5.3738
Wine Gallon.	0.001133331	882.3468	.00339999	294.1156	231.0000	17 1498	15.1990
Ale Pint.	0 .0074325	134.6418	.0222808	44.8806	35.2500	6.6993	5.9370
Ale Gallon 282.	0.000928366	1077.1595	.0027851	359.0533	282.0000	18 9487	16.7960
Ale Barrel.	0.000029011	34469 7315	0000087034	11489.9105	9024.0000	107.1910	95.0000
Beer Barrel.	0.000025788	38777.6124	.0000077364	12925.8708	10152.0000	113.6920	100.7580
Ale Pint at 288.	0 .0072796	137.5392	.021811	45.8464	36.000	6.7710	6 0000
Ale Gallon 288	0.000999999	1000.087070	0. 00.2[illegible]39	366.69569	288.000	19.1493	16.9710
Ale Barrel 288.	0000.28405	35203.5666	.000085212	11734.5222	9216.000	108.326	96.0000
Beer Barrel 288.	0 000.27858	39359.8224	.000076213	13119.9408	10368.000	114.891	101.823
Corn Gall. 272.	0. 000.96156	1039.75572	.00.28852	346.58524	272.250	18.6168	16.500
Corn Buſhel.	0 .000.12026	8321.1228	.00036050	2773.7076	2178.000	52.666	46.668
Coal Buſhel.	0..000.11644	8586.7500	.00034934	2862.2500	2246.060	53.500	47.395

The Numbers in this Table are thus made.

The Numbers in the ſixth Column, are the Number of Cube-Inches contained in any of theſe Veſſels. Thus: A Wine Gallon is 231 Cube-Inches, an Ale or Beer Gallon contains 282, (or 288, as ſome ſay,) the laſt Column are Square-Roots of the ſixth Column, and the Gauge-Points for Square Veſſels.

The Column of Gauge-Points for round Veſſels, are the *D*iameters of Circles, whoſe Superficial-contents are equal to the Cube-Inches contained in thoſe Veſſels, found by the 8*th* Section of Chap. XII.

The Column of *D*iviſors, next the Cube-Inches, are only the Square of the Gauge-Points for round Veſſels, and found by multiplying every particular Gauge-Point for round Veſſels by it ſelf.

The Column of Multiplicators, are the Arithmetical Complements of the *D*iviſors, found thus: The extent from 1 to any *D*iviſor, being laid the contrary way from the ſame 1, gives the Multiplicator.

Thus: the extent from 1 to 36.766, being laid the contrary way from the ſame 1 backward, gives .0272, the *D*iviſor and Multiplicator for a Wine Pint.

But by the Logarithms thus found:

Seek the Logarithm of 36.766, which is 1.56544, 63829. The Arithmetical Complement, is 8.43455, 36170, (to make every Figure 9.)

This ſought in the Logarithms, the natural Number to it is 27199 and more, and therefore .0272 is rather uſed. The adding of .0 is [illegible], ruled by the proper Characteriſtick.

The firſt Colum of Multiplicators, are the third part of the third Column of Multiplicators.

The ſecond Column of Diviſors, are triple to the fourth Column of Diviſors.

CHAP. XVI.

To find the wants in any close Cask, by Numbers and Segments.

FOr this purpose, there be made several particular Lines fit for a Barrel, a Kilderkin, a Firkin, a Pipe-Hogshead, or a Butt, or for any usual Cask, made and sold by *John Brown* in the *Minories*, and other Mathematical-Instrument-Makers.

The use of which, is to put down right into the Bung-Hole, to the opposite Staff, and so far as the Rule is wet, on the proper Line for that Vessel, it shews how many Gallons is in the Vessel; or the measure from the inside of the Bung-Hole, to the Superficies of the Liquor, sheweth how many Gallons will fill the Vessel. But this being too Mecanick for Ingenuous Men, and sometimes by the diversity of Cask, may fail too much in exactness; therefore this way by Segments is more universally useful, and ready enough for all occasions that may occur.

For which purpose, on the printed Rule, is three Lines of Segments, one for a Cylender, the other for a Sphere, the other between for a Spheroid, and by the three, by consideration you may fit all Casks, that of a Cylender serving all Cylender-like Vessels, that of a Sphere for all great swelling Vessels, and that of a Spheroid all ordinary Vessels, which mean Line is put on Four-foot Gauging-Rods, by the Numbers, and used thus:

The dry or wet Inches of any Vessel given, together with the whole Content and Bung *Diameter*: to find the Gallons wanting, to fill the Cask, or how many Gallons is in it.

The

The Rule is

1. The extent on Numbers, from the Bung-*Diameter* to 100 on the Segments, ſhall reach the ſame way from the Inches wet or dry, counted on the Numbers to a fourth Number on the Line of Segments, which remember.

2. Then the extent from Unity, to the Veſſels whole content in Wine or Ale Gallons, counted on Numbers, that extent laid the ſame way from the Number remembred, counted on Numbers, reaches to the Gallons in it, or wanting, to fill the Veſſel.

Example. Suppoſe a Graves Hogſhead, whoſe full content is 63 Wine Gallons, and his Bung-*Diameter* 25 Inches, how many Gallons and Parts are wanting when three Inches are dry, the Axis being placed parrallel to the Horizon?

1. The extent from 25 counted on Numbers, to 100 on Segments, reaches the ſame way from 3 on Numbers to .056 on the Line of Segments, which fourth Number remember.

2. The extent from Unity to 63, the whole capacity on Numbers, reaches the ſame way from .056 the Number remembred counted on Numbers, to 3 53 the Gallons wanting at 3 Inches dry.

By the Pen thus:

Multiply the dry Inches by 100, by adding of two Cyphers thereunto, thus 300: then divide this Product by the Bung-*Diameter*, *viz.* 25, and the Quotient is .12; then 12 ſought on the Line of Numbers, juſt againſt it on the Segments is a fourth Number to remember, *viz.* .056.

Then this fourth Number .056, multiplied by 63, the Veſſels whole capacity, the Product is 3.528, the Wine Gallons wanting at three Inches dry, in a Graves Hogſhead 25 Inches at the Bung, and containing when full 63 Wine Gallons.

On the contrary.

If the Bung-*Diameter* of a Veſſel be 25 Inches, and the

the whole capacity 63 Wine Gallons, when 3 Gallons and 53 Parts are wanting, what ſhall be the number of dry Inches.

1. The extent from the Veſſels whole capacity 63 Gallons to unity, ſhall reach the ſame way from the Gallons wanting, *viz.* 3.53, to 562 the Tabular Segment on Numbers, which keep.

2. Then the extent from 100 on the Line of Segments, to 25 the Bung-Diameter, counted on Numbers, ſhall reach the ſame way from the fourth Number kept .05.62 counted on Segments, to the Inches dry, being counted on Numbers, *viz.* 3 Inches as before. In ſhort thus:

1. As 63 to 1, ſo is 3.53 to 5.62.

2. As 100 on Segments to 25 on Numbers :: ſo is .0562 on Segments to 3 on Numbers.

By the Pen work thus:

Divide the Gallons wanting, by adding of Cyphers to it, by the Veſſels whole capacity, and the Quote is the Tabular Segment.

Then ſeek this Number on the Line of Segments, and juſt againſt it on the Line of Numbers is a fourth, which keep.

Then multiply this Number found by the Bung-Diameter, and the Product is the dry Inches, when two Cyphers are taken away.

Thus: 3.53 the Gallons wanting, with 3 Cyphers added, *viz.* 3.53000, divided by 63 the Veſſels whole capacity, the Quote is 5.62.

Then 562 ſought on the Segments, on the Numbers is 12.

Then 25 the Bung-Diameter, multiplied by 12 the Number found, the Product is 300, which divided by 100, by taking away two Cyphers, remains 3 the dry Inches required.

Four useful Problems for Coopers, to make Cask to any size required, per M. D.

I. The length, content, and Bung-Diameter given, to find the Head-Diameter as a Spheroid.

Multiply the content by 1077.16, and divide the Product by the length, and from the Quote take twice the Square of the Bung-Diameter, the Square-Root of the Residue is the Head-Diameter.

Example. In a Beer Barrel Bung-Diametr 22.7, Head-Diameter 20.2, Length 27.7, Content 37 Gallons.

By the Line of Numbers thus:

1. As 1 to 1077.16, so is 37 the Content to 3985.

2. As 27.7 to 1, so is 3985 to 1439, a seventh; from which taking 1030.58, twice the Square of the Bung, rest .408.42, whose Square-Root is 20.2, the Head-Diameter.

II. The Head-Diameter, the Length and Content given, to find the Bung-Diameter.

Multiply the Content by 1077.16, and divide the Product by double the Length, and from the Quote take half the Square of the Head-Diameter, then the Square-Root of the Residue is the Bung-Diameter.

Example. By Numbers, for the same Barrel.

1. As 1 to 1077.16, so is the Content 37 to 3985 a fourth.

2. As 55.4 the double length to 1, so is 3985 the fourth, to 719.8 a seventh; from whence taking 204.02, half the Square of the Head-Diameter, rest 515.78, whose Square-Root is 27.7, the Bung-Diameter.

III. The Head and Bung-Diameter, and Content given, to find the length of the Vessel.

Multiply the Content 37, by 1077.16, being thrice the Square of the Gauge-Point, and divide the Product by

by twice the Square of the Bung, and once the Square of the Head-Diameters in one Sum, the Quote is the length required.

Example. By Numbers.

1. As 1 to 1077.16, ſo is 37 the Content, to 3985.

2. As 1438.62, the Sum of twice the Square of the Bung, and once the Square of the Head-Diameter to 1, ſo is 3985 to 27.7.

IV. The Bung and Head-Diameters, with the length given, to find the Content.

Multiply the Square of the Head-Diameter, and twice the Square of the Bung, in one Sum by the length of the Veſſel; then multiply the Product by .0009283 .66, the Complement Arithmetical of 1077.16, and the Product is 36.9792315.

Example. By the Line of Numbers.

1. As 1 to 1438.62, the Sum of twice 51529, and once 408.4, the Square of the Head and Bung-Diameters, ſo is 27.7 to 39838, a fourth.

2. As 1 to 92838 the Arithmetical Complement of 1077.16, ſo is 39838 the fourth to 36.97923, the Content required.

CHAP.

CHAP. XVII.

To gauge a great open Square Veſſel in whole or in part.

BY the Rule in Prob. 1, or 2, Chap 11, multiply the length by the bredth, and the Product is the Superficial-content of the Baſe or Bottom in Superficial-Inches, the Dimentions being taken with Inches and Tenths.

Then this Product multiplied by the depth in Inches, gives the Solid-content of the Veſſel in Cubical Inches.

Then this laſt Product divided by 282, the Quote is Ale or Beer Gallons.

Or to avoid Diviſion, Multiply the laſt Product by .0035461, the Product is Ale Gallons, being the Arith. Comp. of 282.

The like Multiplicator for Beer Barrels, is 0000985-0279; or for eaſe ſake in Multiplication, uſe only .0000985.

The like Multiplicator for an Ale Barrel, is 00011-08153, or for brevity 000111.

Example.

There is a Square Tun whoſe length is 218 Inches, and the bredth 171.2 Inches, the Product or Rectangle is 37321.6; then this Product divided by 282, or multiplied by 0035461, (the Arithmetical Complement as before ſhewed in making the Tables,) gives the capacity of one Inch deep in Beer Gallons, *viz.* 132.1391.

Then

Then this laſt Product multiplied by the depth of the Liquor in Inches, gives the Solid-content or capacity in Ale Gallons.

But if you were to find the content in Beer Barrels.

Then multiply the Area of the Baſe, or the Rectangle of the length and bredth, 37322 by 985, and the Product is 3 Barrels, 674217 Parts of a Barrel, contained in one Inch deep.

Laſtly, This Product 3.674217 being multiplied by 36 Inches the depth of the Tun in Inches, the Product is 132 Barrels, 271812 Parts of a Barrel.

By the Line of Numbers, work thus:

As 1 to 171.2, the bredth, ſo is 218 the length to 37322, the Area in Inches.

Then as 282 to 37322, ſo is 1 to 132.39, the Gallons in one Inch deep.

Or ſo is 36 to 4768, the Gallons in 36 Inches deep, the whole Content.

Or to find the Content in Barrels.

As .10152, the Cubical Inches in a Beer Barrel, to 37322; ſo is 1 to 3.674, the Barrels in one Inch deep.

And ſo is 36 the whole depth in Inches, to 132 Barrels 272 Parts.

Laſtly, By the ſide of the Square equal, and Gauge-Points, to find the Solid-content of any Square-Tun in Gallons, Ale or Beer Barrels.

Firſt, Find a mean Proportion Geometrical, between the length and bredth of the Veſſel, which mean Proportion is the ſide of the Square equal, by the ſecond Chapter, being the middle between the length and bredth on a Line of Numbers. Thus the exact middle between 171.2 and 218, is at 123.3.

Then the extent from 16.796, the Gauge-Point for an Ale Gallon in Square Veſſels, to 193.3, the ſide of the Square equal, being twice repeated from 1, gives 132.4; or being twice laid from 36, the whole depth, gives 4768 the whole Content in Gallons.

Again,

Again, The extent from 95, the Gauge-Point for an Ale Barrel in ſquare Veſſels, to 193.3, the ſide of the Square equal, being twice repeated from 1, gives 4.13, the Ale Barrels in one Inch deep.

Or, The ſame extent laid twice from 36, the whole depth, gives 149 Ale Barrels.

Alſo note, The ſame extent laid from any propoſed number of Inches depth of Liquor, two times the ſame way, gives the Solid-content in Wine or Ale Gallons, in Ale or Beer Barrels, as the Gauge-Point uſed ſhall be.

CHAP. XVIII.

To meaſure a great ſquare Taper Tun.

THis is meaſured as the former ſquare Tun, after you have gained a mean ſide of a Square, between the top and the bottom, or the greater and leſſer Squares, thus: as in *Prob.* 12. *Chap.* 11. is fully ſhewed, in brief thus: For the whole Tun, add the greater and leſſer ſides of the Squares at top and bottom together, to find the Sum, and half Sum, and difference.

Example.

Let a great ſquare Taper Tun be propounded, whoſe two ſides at the top are 108, and 120 Inches, and at bottom 130 and 144 Inches, and 30 Inches deep.

The middle between 108 and 120, on the Line of Numbers, is 113.8, being the Square-Root of the Product of 108, multiplied by 120.

Alſo: The middle between 130 and 144, is 136.82, the Square-Root of the Product of 130, multiplied by 144.

Thus: 113.8 is the ſide of a Square equal to the leſſer Baſe.

And 136.8 is the ſide of a Square equal to the greater Baſe.

Alſo: The half Sum of 113.8, and 136.8, is the ſide of a mean Square, between 113.8 the leſſer ſide, and 136.8 the greater ſide, *viz.* 125.

Subſtract alſo 113.8, the ſide of the leſſer Square, from 136.8, the ſide of the greater Square, and the difference is 23, and the half difference is 11.5.

1. Then the extent from the Gauge-Points for Gallons or Barrels, to 125.3, the ſide of the mean Square equal, being twice repeated from the depth, gives the near content in Gallons or Barrels, as the Gauge-Point uſed is.

2. The extent from the ſame Gauge-Point to 11.5, half the difference of the Squares, ſhall reach from 10, one third part of the depth, to a fourth, to add to the former; which is the whole Solid-content.

Example.

1. As 16.796, to 125.3; ſo is 30 twice to 1671.
2. As 16.796, to 11.5; ſo is 10 twice to 4.70: the Sum of both is 1675 Gallons 70 Parts.

Or thus by the Pen:

Find the Rectangle or Product of 108, multiplied by 120, and note it.

Alſo, the Product of 144 by 130, and note that alſo.

Alſo, The Product of 113.84, multiplied by 136.82, the ſides of the Squares equal, to the greater and leſſer Rectangles at each end.

Thus

Thus the Product of 108 multiplied by 120, is equal to 12960
And 144 by 130, is 18720
And 136.82 by 113.84, is 15576

The three Sums added, is 47256
One third part of this Sum, is 15752

Being the Cubical Inches contained in one Inch deep.

Then lastly, This third part multiplied by 30, the Tuns depth, the Product is 472560 cubical Inches in the Fruſtum Pyramid or Tun; which being divided by 282, the Inches in one Gallon, the Quote is 1675 Gallons, and 210 Inches over.

This operation is ſoon done by the Line of Numbers (but it may be not ſo exact) and as a trial of your work by the Pen:

1. As 1 to 120, ſo is 108 to 12960
2. As 1 to 130, ſo is 144 to 18720
3. As 1 to 113.84, ſo is 136.82 to 15576

The Sum of the three Products 47256
One third part of it is 15752

The Inches in one Inch deep, multiplied by 30, the Inches deep is 472560 cubical Inches.

Then as 282 to 1, ſo is 472560 to 1675 Gallons, 210 Inches over.

Laſtly, As 36 to 1, ſo is 1675 to 46 Barrels $\frac{1}{2}$.

To give the Content of a ſquare Taper Tun at one Inch deep, called Inching the Tun, or at every ſix Inches, or as you pleaſe.

In doing whereof it is needful to uſe a convenient order and method, as thus:

1. By the difference of Squares or Diameters at top and bottom, find the difference in 1 or 6 or 4 Inches deep, thus:

Divide the difference of Diameters or Squares, by the whole depth in Inches, the Quote is the difference of Diameters in 1 Inch deep, and ſix times ſo much is for 6 Inches, three times ſo much for 3 Inches, *&c.*

Or, The extent from the depth to the difference of Squares or Diameters, at top and bottom, ſhall reach from 1 to the difference at 1 Inch, or from 2 to the difference at 2 Inches, or from 3, 4, 5, or 6, to the difference of Diameterss at 3, 4, 5, or 6 Inches deep, called *Addend*, when you proceed from the leſſer end to the greater; or *Subducend*, when you proceed from the greater to the leſſer.

2. Find the true drip or fall of the bottom of the Tun, done by putting Liquor in, till the higheſt part of the bottom be juſt covered, or by a Level: then half the Liquors depth in the deepeſt place, is the Tuns drip.

Or, If the Tun be full, the mean difference between the leaſt and greateſt depth, is the true depth to be uſed in Inching the Tun.

3. For ſquare Taper Tuns, find the ſide of the mean Square, between the length and breadth at the leſſer end, then to that add the *Addend* gradually, due to 1 Inch, 2 Inches, or 3 Inches, as you intend to proceed.

Or, Subſtract the ſame accordingly, if you begin at the great end.

4. Note. Theſe Diameters or Sides of the Squares are beſt taken with a long ſliding Rule, or by 2 or 3 Four-foot Rules put together, as at the beginning is hinted, with a Braſs ſocket, which ought to be taken in ſeveral places, to find the mean of all the extreams, and in that place to take your Diameters from top to bottom, horizontally from Inch to Inch.

Againſt every particular Diameter, taken in the middle of every marked Inch, or as it increaſeth or diminiſheth by the *Addend* or *Subducend*, being gathered orderly in a Table, ſet down the Area of that Diameter in Gallons or Barrels as you pleaſe, until you come

come to the top or bottom from your greater or leſſer Diameter, according as you firſt began.

And in ſo doing, you reduce the Taper Tun into ſo many Squares or Cylenders, and the Sum of all thoſe Squares or Cylenders is the Tuns whole capacity,

6. The Areas of theſe particular Diameters, are had by inſpection for all round Tuns, by *Oughtreds* Gauge-Line and ſmall Inches, or by the Cyclometrical Table in ſeveral Books, as *John Smith* in 1673, by *William Hunt* in 1674, by *Richard Collins* in 1676.

Or you may find any of them by the Line of Numbers ſufficiently exact, in this manner following, for round or ſquare Veſſels.

The extent from the Gauge Point (for Wine or Ale Gallons, Ale or Beer Barrels, for round or ſquare Veſſels, according to the Table of Gauge-Points, and Diviſors, and Multiplicators before ſaid) to the Diameter of any Circle, or ſide of the Square equal, being twice repeated the ſame way from the depth of any Veſſel, gives the capacity in Wine or Ale Gallons, Ale or Beer Barrels, and 100 Parts according to the Gauge-Point made uſe of.

Example. In this ſquare Veſſel, at every Inch from 1 to 30 Inches, the depth, the ſquare equal to 108 and 120, the two ſides at the leaſt end, is 113.84, and the ſquare equal to 144 and 130, the two ſides at the greater end, is 136.82, the difference is 22.98 Inches.

Then as 30 the depth to 22.98, the difference of Squares at top and bottom, ſo is half an Inch the middle between the Brim and $\frac{1}{2}$ Inch deep, to 0 383 to add to 113.84, which makes 114.223 for the ſide of the Square equal, at half an Inch from the top, *viz.* in the middle of the firſt Inch deep, counting downward from the leſſer end at top toward the bottom.

Again, The extent from 30 the depth, to 22.98, the difference of the ſides of the Squares equal at top and bottom, ſhall reach from 1 to 0.7664 the *Addend*; then this added to 114.223, makes 114.9914 for the

Square equal at one Inch and a half from the top downward, and ſo for all the reſt, as in the Table following you ſee.

Where note, that you may find every fifth Number by the Line, and the intermediate Numbers by Addition, to prevent miſtakes in Addition.

Inch

Inch. Deep.	Tuns Squ. Equal.		Tuns in Ale Gall.	Area Gall.	Tuns in B. Barr.	Area Barr.
			Gall.	Parts.	Bar.	100 p.
0	113	840	46	28	1	288
0½	114	223	46	90	1	300
1½	114	989	47	60	1	318
2½	115	755	48	25	1	338
3½	116	521	48	90	1	357
4½	117	287	49	60	1	376
5½	118	053	50	20	1	391
6	118	810	50	90	1	414
7	119	570	51	40	1	427
8	120	330	52	20	1	450
9	121	110	52	80	1	467
10	121	885	53	20	1	478
11	122	650	54	10	1	500
12	123	400	54	70	1	520
13	124	160	55	50	1	540
14	124	968	56	20	1	560
15	125	730	56	72	1	576
16	126	480	57	35	1	591
17	127	260	58	15	1	616
18	128	030	58	90	1	636
19	128	790	59	57	1	655
20	129	550	60	24	1	676
21	130	320	60	90	1	691
22	131	080	61	58	1	710
23	131	840	62	40	1	733
24	132	630	63	00	1	750
25	133	380	63	80	1	770
26	134	150	64	46	1	789
27	134	820	65	20	1	809
28	135	680	66	00	1	830
29	136	440	1677	00	46	526
30	136	820				

These Tables were made by a Line of Numbers on a Two-foot Rule, and may differ in the exactneſs of them: if you take not great care in taking the Dimenſions, your errour may be much more, though you work any other way.

Thoſe Numbers in the Table, by the Line are thus had:

As 16.796 the Gauge-Point for a ſquare Tun, to 114.223 the ſide of the ſquare equal; ſo is 1 twice repeated to 46 Gallons 28 Parts, and ſo for all the reſt.

Or, As 100.758 the Gauge-Point for a Beer Barrel, to 114.22 the ſide of the Square equal; ſo is 1 twice to 1 Beer Barrel and 288 Parts, *&c.*

1. Or without finding the Square equal thus: As the Gauge-Point for ſquare Veſſels, to the bredth, ſo is the length to a fourth.

2. As the Gauge-Point to the fourth, ſo is the depth to the Solid-content as the Gauge-Point was.

The difference between theſe particular Sums done by the Line of Numbers, is above 1 Gallon in 46 Barrels, a fault not conſiderable in this matter.

Thus much for ſquare Taper Tuns.

CHAP.

CHAP. XIX.

To Gauge and Inch a round Conical Tun, or Fruſtum of a Cone.

TO meaſure the whole Cone, or Fruſtm Cone, or Tun, is fully ſhewed in *Prob.* 12. *Chap.* 11.

To Inch this Tun, in brief thus:

With a ſliding Rule find the place where the Diameter is a mean, between the leaſt and greateſt, both at top and bottom.

In that place, or in two places ſo found, make Perpendicular Lines down the ſides of the Tun, oppoſite one to the other.

Then divide thoſe Lines into Inches, or every 2, 4, or 6 Inches, or Feet, having due reſpect to the Perpendicular Altitude, and drip or fall of the bottom of the Tun.

Then by the ſliding Rule, take the true Diameter in in the middle, between every mark made on the ſide of the Tun, and ſet it down in a Table; then right againſt every Diameter, ſet down the Area found by the ſmall Inches and Gauge-Line, as you ſee done in the former Example of the ſquare Tun, and the Sum will prove the whole capacity of the Tun, and the particular Sums the capacity in Gallons, at every Inch, or 3, or 6, or 12 Inches deep.

Example. Let the greater Diameter be 144 Inches, and the leſſer Diameter 108 Inches, and the depth 36 Inches.

Firſt, For the Tuns whole content, the eaſieſt and brieſeſt ways.

1. Square the Sum of the Diameters, and note the Product.

2. Mul-

2. Multiply the Diameters one by another, and ſubſtract this Product from the former, and note the Remainder.

3. Multiply this Remainder by the depth, and note the Product.

4. Laſtly, Multiply this laſt Product by 26188, the Arithmetical Complement of 3.81972, the triple of 127324, the Diviſor in the Table, the Product is Cube-Inches.

Or, By .00113331, the Arithmetical Complement of 882, the triple of 294.1156, the Diviſor for a Wine Gallon, then the Product will be Wine Gallons.

Or, By .00092836*66* for Ale Gallons, *ditto.*

Or, By .00002578*97* for Beer Barrels.

Or, By .00002901146 for Ale Barrels, being the Arithmetical Complement of triple the Diviſor for an Ale Barrel, *viz.* 11489.9105, being the Square of the Gauge-Point for round Veſſels, this is one way by the Pen.

The operation thereof by the Line, as an example thereof.

The extent from 1 to 108, reacheth from 144 the ſame way, to	15552
The Square of 252, the Sum is	63504
The Rectangle 15552 taken from the Square of the Sum of Diameter, reſts	47952
This Remainder multiplied by 36	36
The depth the Product is	1726272

Then the extent from 1 to 1726272, laid the ſame way from 26188, gives 452076 cubical Inches.

Or, The ſame extent laid from 00113331, gives 1957 Wine Gallons.

And the ſame extent laid from 00092836*66*, gives 1602$\frac{1}{2}$ Beer Gallons.

And the ſame extent laid from 00002901146, gives 50 Ale Barrels and $\frac{1}{10}$.

And

And the ſame extent laid 0000257879, gives 44 Beer Barrels, and 53 Parts of 100.

A ſecond way (as in *Prob.* 12.) is to meaſure the whole Cone, and the Cone cut off.

Firſt, For the whole Cones Altitude, ſay:

As 36, the difference of Diameters, is to 108 the leſſer Diameter: ſo is 36 the Tuns depth, to 108 the Altitude of the Cone cut off; to which adding 36 the Tuns depth, gives 144 the whole Cones Altitude.

Then ſquare the Diameter, and multiply the ſquares by one third of the Cones Altitude, then divide that Product by the ſquare of the Gauge-Points in the Table, or multiply it by the Multiplicators there, and the Quote or Product ſhall give Inches, Gallons, or Barrels, as the Gauge-Point uſed ſhall be.

This work is to be done for both Cones, the leſſer and greater, and the remainder after Subſtraction is the anſwer.

Or by the Numbers thus.

The extent from the Gauge-Point to the Cones Diameter at Baſe, laid twice from a third part of the Altitude, gives the Content in Inches, Gallons, Barrels, as the Gauge-Point uſed was.

Then the leſſer Cone taken from the greater, remains the capacity of the Fruſtum Cone or Tun required.

A third way eaſie to remember, after the manner of a Cylender.

Add the greater and leſſer Diameters together, to get the Sum and half Sum, and ſubſtract one from the other, to find the difference and half difference together, with one third part of the depth.

1. Then the extent from the Gauge-Point to the half-Sum of the Diameters, being twice repeated from the depth, gives the near Content in Gallons or Barrels, as the Gauge-Point was.

2. The

2. The extent from the same Gauge-Point to the half-difference of Diameters, being twice repeated from the third part of the depth, gives a number to add to the first, to find the full true Content of the Conical Tun required.

The operation by the Pen is thus:

1. The Square of 126, the half-Sum is 15876, this multiplied by .0027851, the number in the Table for Ale Gallons, is 44.2162476, this multiplied by the depth 36, is 1591.7249.

2. Then the Square of 18, the half-difference of Diameters, is 324, and the Multiplicator multiplied by this produceth 9023724; and this Product multiplied by 12, one third of the depth, gives 10.8284, which added to 1591.7249, the first Product makes 1602.5533 Ale Gallons, as before.

Or for a suddain trial by the Line of Numbers, a small addition to the half-Sum of Diameters, will give the near Content at one operation.

Thus in four Examples:

1. Diameters 144 and 108, deep 60 Inches.
   As 18.9468 to 126, more 4 Tenths:
   So is 60, twice to 2671 Gallons.

2. Diameters 215 and 209.6, deep 54 Inches.
   As 18.9468 to 212.3 the half-Sum, more 05 ?
   So is 54 the depth, to 6778 Gallons.

3. Diameters 72 and 64, depth 30 Inches.
   As 18.9468 to 68, more 04 Tenths of a Tenth:
   So is 30 the depth, to 386.796 Gallons.

4. Diameters 40 and 36, deep 20.
   As 18.9468 to 38, more $\frac{03}{100}$ of an Inch in 100:
   So is 20 the depth, to 80 Gall. 508 Parts of 1000.

By

By these Examples you may see that a small matter added to the half-Sum of Diameters, gives a true equated Diameter, to work it at one operation.

And note, that the Tenths of an Inch, or 100 Parts of an Inch, to add, are more or less, according to the difference of the Diameters of the Tun.

Also: By the Gauge-Line and small Inches, you may by inspection see, the quantity of Gallons contained in one Inch deep, at any Diameter to 3 Figures, under 100 Inches Diameter, or to two Figures to 800 Inches Diameter, that is to say, to Gallons, and 100 Parts, to 100 Inches Diameter, and to single Gallons to 800 Inches Diameter.

Therefore the plainest way of Inching any round Tun, is by a sliding Rule, and the Gauge-Line on it, if you please: for so have I made some, to extend to 11 Foot, that having found the place of the mean Diameter of your Tun or Copper, whether Conical or Spheroidal, taking the Diameters in the middle of every Inch deep, there you have the solid capacity of every particular Inch, and the Sum of all is the whole Content; or the Sum of any part, is the Content of so many Inches deep.

Note, That in Tuns that stand leaning, there must be consideration for the drip or fall of the bottom.

Also in Coppers, to abate for the rising Crown, as before is shewed in measuring the Segments of a Globe.

CHAP. XX.

To meaſure a Spheroidal Tun, or Brewers Copper at once.

FInd the Diameter at the top, with a ſliding Rule, in ſeveral places, to get the mean Diameter.

Meaſure the Diameter at bottom alſo, in like manner, juſt above the top of the riſing Crown.

And the true depth from the top of the Crown, to the brim of the Copper; and alſo the Altitude of the riſing Crown, and the Copper Diameter at the bottom of the Copper.

As thus for Example:

Let *a*, *b*, *c*, *d*, repreſent a Brewers Copper, *a*, *b*, the bredth at the top 147.5 Inches, the Diameter at the very bottom *c*, *d*, 109 Inches, the Diameter at the top of the Crown 123.5 Inches *g*, *h*; the depth *e*, *f*, 34.5, but the depth *e*, *i*, 45.5, and the Altitude of the Coppers Crown 11 Inches.

Firſt, Find the Area of the Spheroid *a*, *b*, *c*, *d*, as if it had no riſing Crown, thus:

Juſt againſt 147.5, counted on ſmall Inches, is in the Gauge-Line, which ſet down twice	5 60.5933 2 60.5933
Juſt againſt 109, is	33.0898
Then theſe three Sums added, are	154.2764
And mult. by one third part of the depth	15.16
Produceth	2338.8302

Then

Then from this Sum, deduct the rising Crown, found as before in *Prob.* 11, or readily thus:

Right against 109 on small Inches, on a Gauge-Line for Ale, is	33.0898
Which multiplied by 5.625, the half Altitude, more 1.8.	5.625
Produceth	186.1256

Then this taken from the former Sum, remains 2152 Gallons 70 Parts, the Coppers whole Content.

Which Work for a suddain trial, may be done at one operation, and near the truth, thus:

Count the Diameter in the middle of the Coppers Altitude, as a mean Diameter, as here in this Spheroid Copper, the middle Diameter will be about 131 Inches.

Then the extent from 18.95, to 131, being twice repeated from 45, the Coppers whole depth, gives 2150, two Gallons more than before.

Or, The extent from 113.69 to 131, laid twice from 45.5, the whole depth, gives near 60 Barrels for the whole Content.

But note, when the sides are straight, then it is to be counted Conical.

Then the extent from the Gauge-Point 113.69, to 128.25, the half-Sum of the Diameters, being twice repeated from 45.5 the whole depth, gives 58 Barrels 95 Parts; from which deducting the Coppers Crown, being 5 Barrels and 18 Parts, remains 53 Barrels 77 Parts. Such difference is between a Spheroid and a Conical Vessel, when great, as here, 6 Barrels and more.

The Inching of this Copper is best done as before, by taking the true Diameter in the middle of every Inch, marked down the Copper side, in that place where the mean Diameter is found to be.

CHAP. XXI.

Four useful Problems for a Gauger to know.

PROB. I.

Having the Diameters and Length, to find the Diagonal-Line.

ADD the Square of the half-Sum of both the Diameters, to the Square of the half-length; then the Square-Root of the Sum, shall be the Diagonal-Line required.

Example. If the half-length be 20, and the Square thereof 400, the half-Sum of 32 and 24, the two Diameters is 28, the Square thereof 784; then the two Squares added, make 1184, whose Square-Root is 34.408, the Diagonal-Line required.

PROB. II.

Having the Diameters and Diagonal, to find the true length of a Cask.

Substract the Square of the half-Sum of both Diameters, from the Square of the Diagonal; then the Square-Root of the Remainder, is the half-length required.

Example. The Square of the Diagonal-Line 34.408, is 1184; from which when 784 the Square of 28, the half-Sum of 32 and 24, the two Diameters, is substracted, remains 400, whose Square-Root is 20, the half-length.

PROB.

PROB. III.

Having the depth and Diameters of Vessel, to find the length of the slant heighth or depth.

To the Square of the depth, add the the difference of Diameters; then the S that Sum shall be the slant height or leng ing side.

Example. Let the Perpendicular dep difference of Diameters 8; then the *viz.* 400, and the Square of 4, the half added, is 416, whose Square-Root is slant height or depth required.

PROB. IV.

Having the Diagonal, the depth and to find the Diameters.

1. From the Square of the Diagonal, Square of the depth, the Square-Root of der is the half-Sum of Diameters.

2. From the Square of the slant depth, Square of the Perpendicular depth; then Root of the Remainder, is the half-differ meters, which being added to the half-Su ters, giveth the greater Diameter, an from the half-Sum of Diameters, leaveth the Diameters.

Example 1. When 400 the Square of 2 taken from 1184 the Square of 34.408 th the Remainder is 784, the Square of 28 of the two Diameters, by extracting the S

III.

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the half of 8, viz.
Root is 20.396,

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are of 20 the depth,
.408 the Diagon:
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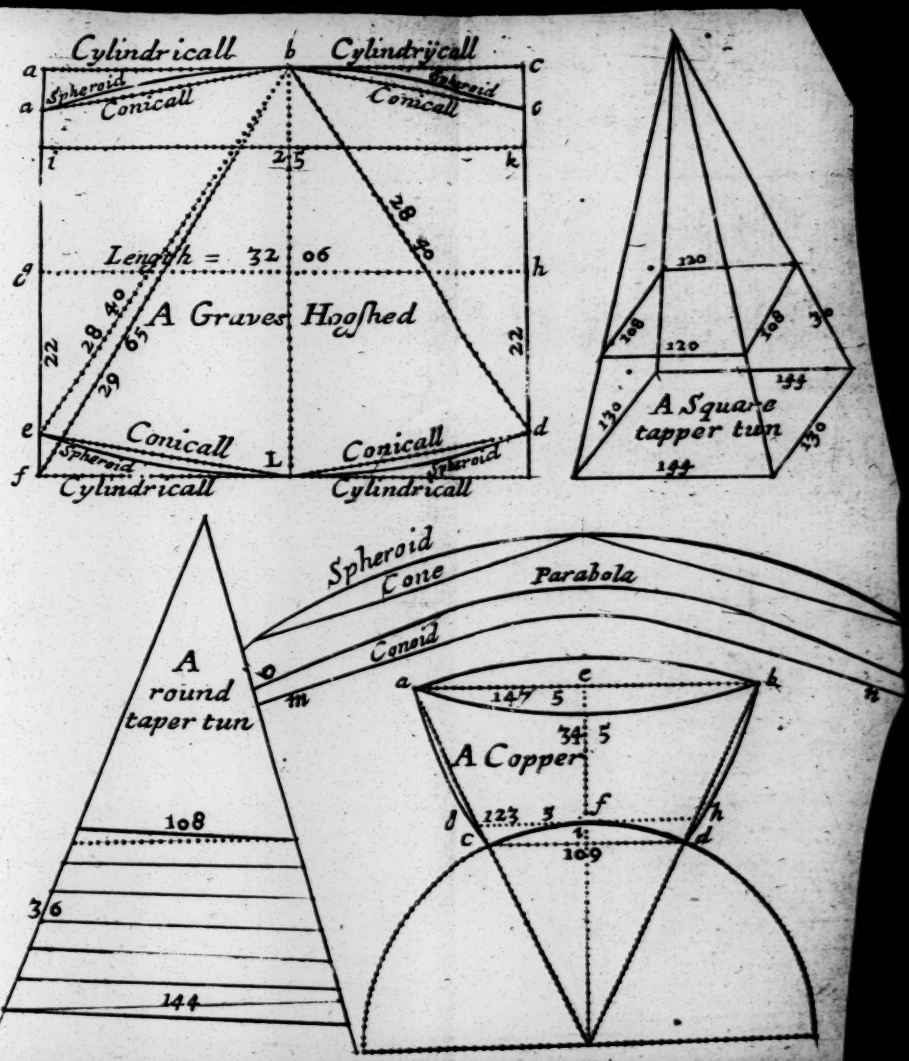
Example



Example 2. The Square of the Perpendicular depth 20, *viz.* 400, taken from the Square of 20.396, *viz.* 416, the ſlant depth, remains 16 the Square of 4; which added to 28, the half-Sum of Diameters, gives 32 the greater Diameter, or being taken from 28, reſts 24, the leſſer Diameter required to be found.

Theſe four Problems may be of good uſe to a Gauger many times, in taking the dimentions of Casks, either cloſe or open.

CHAP.

CHAP. XXII.

By the Line of Numbers to ſupply the want of Tables of Areas of Circles; or to find the Area of any ſquare or round Tun or Veſſel in Wine or Ale Gallons, Beer or Ale Barrels, at one, or any number of Inches deep.

FIrſt, For round Veſſels.

At any Diameter given, to find the true capacity in Wine or Beer Gallons, at once Inch deep.

The extent from the Gauge-Point for Wine or Beer Gallons (round Veſſels) to the Diameter counted on Numbers, being twice repeated the ſame way from 1, gives the capacity required in Wine or Ale Gallons, and 100 Parts.

Example. At 30 Inches Diameter.

The extent from 17.15 to 30, being repeated twice the ſame way from 1, gives 3 Gallons 06 hundred Parts, the true Content at 1 Inch deep.

Or, The ſame extent laid from any number of Inches deep, two times repeated, gives the liquid Content in Gallons at ſo many Inches deep.

Thus: the extent from 17.15 to 30, being twice repeated the ſame way from 8, the Inches deep, gives 24 Gallons 50 Parts, the true Content at 8 Inches deep.

The ſame Work ſerves ſerves for Ale Gallons, or Ale or Beer Barrels, only remember to uſe the right Gauge-Point for Ale or Beer Gallons, Ale or Beer Barrels, as in the Table Page 103, is orderly inſerted.

To

To do the ſame Work by the Pen, do thus:

Square the Diameter given, and multiply it by the Multiplicators in the Tables, and the Product is the Gallons and Parts contained in one Inch deep, as is often done before; then this Product multiplied by the Inches and Tenths deep, gives the whole liquid Content in Gallons or Barrels, as the Multiplicator made uſe on was.

Thus: the Square of 30 is 900, then 00344, the Multiplicator in the Table for a Wine Gallon, multiplied by 900, produceth 3.060 Wine Gallons, the liquid capacity in one Inch deep; and then this multiplied by 8 the depth, produceth 24 Gallons 48 Parts, near as before.

The ſame Work ſerves for Barrels, Ale or Beer, uſing the proper Multiplicator in the Table.

The lower part of the Table is the like Work for the Ale or Beer Gallons, at 288 cubical Inches to the Gallon, or for a Corn or Sea-coal Buſhel.

2. Having the ſide of any ſquare Tun, or the ſide of a ſquare equal in Area, to any irregular Flote, Cooler, or any other Poligon (or many-ſided Figure,) to find the liquid capacity thereof at one or many Inches deep, in cubical Inches, Wine or Beer Gallons, Ale or Beer Barrels, by the Line of Numbers.

The extent from the proper Gauge-Point, according to the intent of the queſtion, to the ſide of the ſquare equal to the Area of any irregular Figure, being twice repeated from 1, ſhall reach to the liquid capacity in Gallons or Barrels, according to the Gauge-Point uſed.

Or, the ſame extent twice repeated from the Inches deep, gives the whole liquid capacity, or Content required.

Example.

Example. If the side of a Square of 263 Inches each side, be equal to the Superficies of a Brewers Cooler; how many Ale Barrels may be contained in one, or 8 Inches and a half deep?

The extent from 95, to 263, being twice repeated the same way from 1, gives 7 Barrels 660 Parts Ale, in 1 Inch deep.

Or, the same extent being turned two times the same way, from 8.5 the depth of Liquor, gives 65 Barrels 110 Parts, the whole liquid Content in Ale Barrels required.

3. Having the Area of any regular or irregular Vessel given in Cubical Inches, to find the Content in Gallons or Barrels, by the Line of Numbers.

Seek in the Table for the number of Cubical Inches contained in one Gallon, Barrel, *&c.* then the extent from the number of Inches found in the Table, to the Inches in the Area given, shall reach the same way from 1, to the Gallons or Barrels in one Inch deep, or the same way from the Inches deep to the whole Content at so many Inches deep.

But if the whole Solidity in Inches be given, then say, The extent from the Inches in the Table, contained in one Gallon, or Barrel to 1, shall reach the same way from the solidity in Inches, to the solidity in Gallons or Barrels required.

Example. Suppose a Vessel contains 4597 Cubical Inches, how many Gallons is this in Ale measure?

The extent from 282 to 1, being laid the same way from 4597, gives 16 Gallons and 31 Parts of a Gallon. By the Pen: divide 4597 by 282, the Quote is 16 Gallons 1.25 Parts.

4. To

4. To find the Exciſe due for any number of Barrels of Beer, with Diſcount for the allowances of 3 Barrels in 23, for Beer, and of 2 Barrels in 22 for Ale, Work thus:

If 23 Barrels of ſmall Beer pay 15 Shillings, what ſhall any greater or leſſer number of Barrels pay for Exciſe?

The extent from 23 to 15, reaches from 60 to 39 Shillings 2 Pence, the Exciſe due for 60 Barrels of ſmall Bear, the allowance being diſcounted.

Again, If 23 Barrels of ſtrong Beer pay 3 Pounds 5 Shillings for Exciſe, what ſhall 60 Barrels pay?

The extent from 23 to 3.25, ſhall reach from 60 to 8 Pounds 9 Shillings 10 Pence near.

Again, If 22 Barrels of ſtrong Ale pay 3 Pounds 25, what ſhall 60 Barrels of the like Ale pay?

The extent from 22 to 3.25, ſhall reach the ſame way from 60 to 8 Pounds 17 Shillings 3 Pence.

To work theſe three laſt Rules by the Pen, do thus by Diviſion, uſing theſe Diviſors, *viz.* 30.666 for ſmall Beer, or 7.076 for ſtrong Beer, or 6.769 for ſtrong Ale, dividing the number of Barrels by the proper Diviſor, and the Quotient ſhall be the Pounds and Shillings due for the Exciſe.

Example.

What Exciſe is due for 48 Barrels of ſmall Beer, 15 Shillings being due for 23 Barrels?

Divide 48 the number of Barrels, by 30.666, when 6 Cyphers are added to 48, and the Quotient ſhall be 1.5652, which reduced to Money by the Line of Pence, is 1 Pound 11 Shillings 3 Pence 2 Farthings 1.5 Farthing, the exact anſwer to the queſtion.

But

But if Multiplication be more easie to you, then use these Multiplicators, being the Arithmetical Complements of the former Divisors, as after the Table of Gauge-Points is formerly shewed, or else made in this manner.

Divide 075, (the Decimal-Fraction of 15 Shillings, the Excise due for 23 Barrels of small Beer) by 23, adding 5 Cyphers to .75, and the Quote is .032609, the Multiplicator for small Beer Barrels, as in the following Table.

Multiplicator.		Divisors.
.032609	Small Beer	30.666
.141304	Strong Beer	7.076
.147727	Strong Ale	6.769

Or for strong Beer Barrels, divide 3 *l.* 25 the Decimal of 3 Pounds 5 Shillings, the Excise due for 23 Barrels of strong Beer, adding Cyphers to 3.25 by 23, and the Quote is .141304, the Multiplicator used for strong Beer Barrels, thus:

Example.

What Money is due for 50 Barrels of strong Beer?

Multiply .141304 by 50, and the Product is 7.0652, reduced thus: The 7 is 7 Pounds, then 0 in the second place says, no two Shillings; then 652 sought in the Line of Numbers, just over it in the Line of Pence, is 15 Pence 2 Farthings and half a Farthing, the exact answer being 7 Pounds 1 Shilling 3 Pence 2 Farthings 1.5 Farthing.

The

The like Diviſors or Multiplicators may you make, for the Exciſe of Wine or Brandy, or Coffee, or any other Liquors.

5. To reduce Ale Gallons to Wine Gallons, or the contrary; or Beer Barrels to Ale Barrels, or the contrary, by the Line of Numbers.

The extent from 231, the Inches in a Wine Gallon, to 282 the Inches in a Beer or Ale Gallon, ſhall reach the ſame way from 90 Beer Gallons, to near a 110 Wine Gallons: Or, the extent from 282 to 231, ſhall reach the ſame way from 11 Wine Gallons to 9 Ale, and near 1.100 Part more.

By the Pen.

Multiply the Wine Gallons by 231, and divide the Product by 282, and the Quote is Ale Gallons.

And the contrary for Wine Gallons.

Or elſe by Multiplication only, to bring Wine Gallons to Ale Gallons, multiply the Wine Gallons given by .81915, the Product is Ale Gallons.

But to bring Ale Gallons to Wine Gallons, multiply the Ale Gallons given by 1.2208, the Product is Wine Gallons.

Thus: 72 Wine Gallons multiplied by .81915, and the Product is 58.9824 Ale Gallons.

Or, 58.9824 Ale Gallons multiplied by 1.2208, the Product is 72 Wine Gallons.

Or, this Work is done by inſpection only, by the two Diagonal-Lines, or the two Gauge-Lines.

Theſe Multiplicators are found by dividing 231 by 282, and 282 by 231 decimally.

To reduce Barrels of Beer to Barrels of Ale. By the Line of Numbers ſay,

As 32 to 36: ſo is 40 Beer Barrels to 45 Ale Barrels.

Or

Or on the contrary, As 36 to 32: so is 90 Ale Barrels to 80 Beer Barrels.

By the Pen.

Having Ale Barrels, to bring them to Beer Barrels.

Multiply 88889 by the number of Ale Barrels, and the Product is Beer Barrels.

Thus: 88889 being multiplied by 90 Ale Barrels, the Product is 80 Beer Barrels.

Or on the contrary, 80 Beer Barrels multiplied by 1.125, the Product is 90 Ale Barrels.

This Multiplicator is thus found.

Add three Cyphers to 36000, thus: and then divide it by 32, the Quotient is 1.125.

Or, Add four Cyphers to 32.0000, and divide it by 36, and the Quote is .88889.

But by the Line of Numbers.

The extent from 32 to 36, being laid from 1, gives 1.125.

And the same extent from 36 to 32, laid the same way from 1, gives 88889, as before.

The Table of Segments is very well and readily supplied by the Line of Numbers and Segments upon every Rule, for Spheroid Segments the most necessary for swelling Vessels, but on the print joyned to the Book is Segments of a Cylender, and Segments of a Globe also, that you may see and discover the difference between them.

The Segments of a Cylender, are the same with Mr. *Edmund Gunters*, 1624.

The Segments of a Sphere were first (as I suppose) done by Mr. *Samuel Foster*, and the Segments for a Spheroid made by *John Brown*, 1668.

Note, that these Segments on Lines, are better than short Tables, for here you may see the proportional part, between one number and another, better and readier by far.

CHAP. XXIII.

Of Corn and Coal Meaſures.

IN the like Tablet of Gauge-Points, you find that a Corn Gallon contains 272.25 cubical Inches, then 8 Gallons being a Buſhel, it muſt contain 8 times as many cubical Inches, *viz.* 2178.

The Sea-Coal Buſhel is 1 Corn quarter greater in Liquid capacity then the Corn Buſhel, and contains 2246.06 cubical Inches, and the breadth of thoſe Sea-Coal Meaſures to be well obſerved to agree with the Sea-Coal Fat in the heap thereof.

The breadth of Sea-Coal Meaſures, from out to out, is as followeth.

	In.	*P.*	*Solid-Cont.*	*uſual depth.*
Fat	44	0	20221.54	13.00
Buſhel	19	5	2246 06	08.36
Half-Buſhel	14	5	1123.03	07.65
Peck	11	5	561.515	06.16
Half-Peck	9	5	280.7575	04.48

The inſides, are to be ſo broad and deep, as to hold the number of Cube-Inches proper to each Meaſure, which will be found to be near as in the Table, in meaſuring whereof you muſt conſider the roundneſs of the Veſſel, and that the bottom be flat, and of equal depth, and to find a mean Diameter, and depth, to do right between Man and Man.

But in Corn Meaſures that be filled full, and then ſtruck even, there is not ſuch regard had to the bredth of

of the Meaſure, ſo as it contain the true quantity of cubical Inches, which by the Line of Numbers is thus perform d.

The extent from the equated Diameter of the inſide of the Veſſel, to the Gauge-Point 18.6168, being twice repeated the ſame way from the Content in Gallons, ſhall reach to the true depth in Inches and 100 Parts.

Exam le for a Corn Buſhel of 18 *Inches broad within.*

As 18 to 18.6168, the ſame extent turned twice the ſame way from 8, the Gallons in a Corn Buſhel gives 8.56, the Inches that ſuch a Buſhel ought to be in depth, for a true Corn Buſhel at 272.25 Inches to a Gallon.

Another Example for the half Buſhel.

The extent from 13 Inches, the Diameter on the inſide of the half-Buſhel, taken at ſeveral places acroſs, to get the mean Diameter to 18.6168 the Gauge-Point, ſhall reach (being twice repeated) the ſame way from 4 the Gallons in half a Buſhel, to 8.21 the Inches deep required.

Again, For the Peck at 10 *Inches and* 5 *broad.*

The extent from 10.5, to 18.6168, laid twice the ſame way from 2, the Gallons in a Peck, ſhall reach to 6.28, the Inches deep.

Again, For half a Peck at 9 *Inches in Diameter within.*

The extent from 9 to 18.6168, laid twice the ſame way from 1, the Gallons in half a Peck, ſhall reach to to 4 278 the Inches deep required.

By the Pen work thus, for Inches:

First, By the given Diameter find the Area in Superficial Inches, by *Rule* 3, in *Prob.* 10. *Chap.* 11. which here at 9 Inches Diameter, is 63.58, the Area of the bottom; then 272.25 the Cubical Inches in a Gallon or half-Peck, divided by 63 58, the Quote is 4.282, near as before, the depth required.

Or, Square the Diameter, multiply it by .78539, the Multiplicator for Inches, and divide the Product by 272.25, the Quote is the depth of the half-Peck required.

But by the Pen for Gallons thus:

Square the Diameter, and then multiply it by .002885 2, the Arithmetical Complement of the Square of the Gauge-Point found, as just after the Table of Gauge-Points is shewed. And the Product is 0.2337012, then 1 Gallon the Content of half a Peck, divided by .0.2337012, the Quote is 4.280, the depth in Inches required.

Again, If the depth and content of a Peck be given, to find the true Diameter or breadth readily by the Line.

Divide the Space on the Line of Numbers, between the length and the content, into two equal parts; the Compasses so set, will reach the same way from the Gauge-Point, to the Diameter of the Vessel required.

Example.

If a Peck be 7 Inches deep, how broad ought it to be to hold just 2 Gallons, at 272 25 Inch to the Gallon?

The half distance between 7, the depth in Inches, and 2, the content in Gallons, being laid the same way from 18 6168, shall reach to 9.95 the breadth.

Note,

Note, That if you measure between 7, the depth, and 2 Gallons the content, on the Line of two Radiusses, and lay the same extent from the Gauge-Point, counted on the single Line to one Radius, it shall give 9.95, the breadth as before.

By the Pen thus: Example as before.

Multiply the Square of the Gauge-Point, for a Corn Gallon, *viz.* 346.58524 (found in the fourth Column of the Table of Gauge-Points and Divisors,) by 2 the Gallons in a Peck, the Product is 693.17048; then divide this Product by the length, *viz.* 7, and the Quote is 99.02449 whose Square-Root is 9.9512, the true Diameter of the Peck, at 7 Inches deep.

Lastly, Any Vessel being propounded, the depth and Diameter given, to find the Content in Corn Gallons.

The extent from the Gauge-Point 18.6168, to the Diameter, being twice repeated the same way from the depth, shall give the content in Corn Gallons.

Example at 18 *Inches broad, and* 8.56 *deep.*

The extent from 18.6168, to 18, being laid twice the same way from 8.56, gives 8 Corn Gallons, the content of a Corn Bushel.

By the Pen.

Square the Diameter, multiply it by the depth, then multiply the Product by 00.28852, or divide it by 346.58524, and the Product or Quote is the content in Corn Gallons.

For the ready and easie measuring of Corn, and Sea-Coal Measures, there is a very convenient ready

Rule, made by *John* and *Thomas Brown*, at the *Sphere* and *Sun-dial* in the *Minories*, whose use is thus:

Measure the Diameter across, from inside to inside, and just there, the near edge of the Measure sheweth how deep it ought to be.

Example.

If a Corn Bushel be 19 Inches broad, apply the Measure right across the Bushel, in the mean breadth, putting the beginning-end of the Rule to the far side, then the near side shall shew 7 Inches 68 Parts for the depth of the Bushel.

But for a Sea-Coal Bushel, it should be 7 Inches 93 Parts, as the next Number forward sheweth.

The like use is for the half-Bushel, Peck and half-Peck, Pottle, Quart and Pint, all on one Rule, at any convenient usual Diameter.

A Table of the Dimentions of several Wine and Beer Vessels used in England, *with their Contents in Gallons and Cubical Inches.*

Names of	Head	Bung.	Leng.	Solid Content.		
Wine Vessels	In 100	In 100	In 100	W.Ga.	A.Gal.	Inches
Ren.W.Fat.	31.15	42.32	70.50	360.00	295.00	83160
Tun of Wine	38.50	42.20	44 00	252.00	206.25	58212
Tun of Oyl	36.00	42.00	40.80	236.00	193.50	54516
Canary Pipe	21.82	28.65	48 50	116.40	95 40	26888
Brand. Hog.	22.70	25.80	33.50	70.00	57.30	16170
A Tercion	23.00	27 00	37.40	84.00	68.80	19404
Graves Hog.	22.42	25.40	31.20	63.34	51.80	14631
H.C.Hogsh.	21.20	23.58	30.82	54.55	44.70	12601
A Terce	16.26	18.30	39.60	42.00	34.30	9702
A Runlet	15.70	17.30	30.70	29.47	24.10	6808
Beer Vessels.						
A Butt	25.00	31.00	46.00	131.80	108.00	30450
A Puncheon	23.60	28.30	36.00	87.80	72.00	20304
A Hogshead	22.00	26.00	32.00	66.00	54 00	15228
A Barrel	20.20	23.00	27.00	44.50	36.50	10152
Kilderkin	16.10	18.50	21.00	22.30	18.30	5076
A Firkin	12 80	14.70	16.60	11.12	9.11	2538
A Gallon	7.70	8.50	5.46	1.22	1.00	282
Ale Vessels.						
A Barrel	19.40	22.00	26.20	39.75	32.55	9024
Kilderkin	15.40	17.60	20.80	20.00	16.40	4522
A Firkin	12.20	14.00	16.40	10.00	8.18	2256
Sea-Coal M.	Top.	Botto.	Depth	W.Ga.	Cor.G.	Inches
A Fat	42.00	45.00	13.60	87.30	74.25	20121
A Bushel	18.54	18.80	8.18	9.70	8.25	2247
Half-Bushel	13.75	13 88	7.50	4.85	4.12	1123
A Peck	10.65	10.75	6.26	2.42	2.06	561
Half-Peck	8 95	9.00	4.44	1.21	1 03	280

FINIS.

The Index, ſhewing the Contents of this Book, and the Pages where to find them readily.

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